Math 085
Spring, 2009

NAMES: Key

radicands (numbers under the radical)

Final Exam
over 1.1 – 9.3

*Show the steps* to get to your answer to receive full credit on a problem.
Each problem is worth 10 points. *Be exact* unless told otherwise.
All ~~variables~~ are non-negative real numbers.
Remember, for each word problem, be sure to label the variable, write one equation for the situation, solve the equation and write a summary sentence!

Simplify each statement:

Factor each polynomial:
(Reminder: Factor out the GCF first and be sure to factor completely!)

1. $6x^3 - 30x^2 - 9x + 45 =$

$3 \cdot (2x^3 - 10x^2 - 3x + 15) =$

$3[2x^2(x-5) - 3(x-5)] =$

$3(x-5)(2x^2-3)$

OR

$6x^3 - 30x^2 - 9x + 45 =$

$6x^2(x-5) - 9(x-5) =$

$(6x^2-9)(x-5) =$

$3(2x^2-3)(x-5)$

2. $36x^2 - 60x + 25 =$

$(6x)^2 \quad (5)^2$

$2 \cdot 6x \cdot 5 = 60x$ ✓

$(6x-5)(6x-5)$

or

$(6x-5)^2$

Factor each polynomial expression:

3. $\frac{4}{3}x^2 + \frac{4x}{1} + \frac{3}{1} =$ $\frac{4}{3}x^2 + \frac{3}{3}\cdot\frac{4x}{1} + \frac{3}{3}\cdot\frac{3}{1}$

$= \frac{4}{3}x^2 + \frac{12x}{3} + \frac{9}{3}$

$= \frac{1}{3}(4x^2 + 12x + 9)$

$= \frac{1}{3}(2x+3)(2x+3)$

or $\frac{1}{3}(2x+3)^2$

4. $2x^2 - xy - 3y^2 =$ $(2x - 3y)(x + y)$

5. Fill in the LCM (Least Common Multiples) and the GCF (Greatest Common Factor) for this pair of expressions:

| | Expressions | GCF | LCM |
|---|---|---|---|
| | $28x^2y^3,\ 42x^4y^2$ | $14x^2y^2$ | $84x^4y^3$ |
| | | | |

$28x^2y^3 = 2 \cdot 2 \cdot 7 \cdot xx \cdot yyy$

$42x^4y^2 = 2 \cdot 3 \cdot 7 \cdot xxxx \cdot yy$

GCF $= 2 \cdot 7 \cdot xx \cdot yy$

LCM $=$ GCF $\cdot 2 \cdot 3 \cdot x^2y = 14x^2y^2 \cdot 6x^2y$

Simplify this expression:

6. $\dfrac{x^2-y^2}{y} \div \dfrac{x^2-xy}{xy} =$

$= \dfrac{x^2-y^2}{y} \cdot \dfrac{xy}{x^2-xy}$

$= \dfrac{(x+y)(x-y) \cdot x \cdot y}{y \cdot x(x-y)}$

$= x+y$

Simplify each expression:

7. $\frac{4x-y}{4x^4y^3}+\frac{x-3y}{3x^3y^4}=$

$\frac{4x-y}{4x^4y^3}\cdot\frac{3y}{3y}+\frac{x-3y}{3x^3y^4}\cdot\frac{4x}{4x}$

$LCD = 4\cdot 3\cdot x^4y^4$

$= 12x^4y^4$

$= \frac{3y(4x-y)+4x(x-3y)}{LCD}$

$= \frac{12xy-3y^2+4x^2-12xy}{LCD}$

$= \frac{(4x^2-3y^2)}{12x^4y^4}$

8. $\frac{2x}{3-x}-\frac{1}{x-3}=$

$\frac{-1}{-1}\cdot\frac{2x}{3-x}-\frac{1}{x-3}=$

$\frac{-2x}{x-3}-\frac{1}{x-3}=$

$\frac{-2x-1}{x-3}$ or $\frac{-1(2x+1)}{x-3}$

OR

$\frac{2x}{3-x}-\frac{1}{x-3}\cdot\frac{-1}{-1}=$

$\frac{2x}{3-x}-\frac{-1}{3-x}=$

$\frac{2x-(-1)}{3-x}=$

$\frac{2x+1}{3-x}$

Simplify each expression:

9. $\sqrt{54xy^6}\sqrt{84x^3y} =$ $\sqrt{9 \cdot 6 \cdot 4 \cdot 21 x^4 y^7}$

$= 3 \cdot 2 \cdot x^2 y^3 \sqrt{6 \cdot 21 y}$

$2 \cdot 3 \cdot 3 \cdot 7$

$= 6x^2y^3\sqrt{3^2 \cdot 2 \cdot 7 \cdot y}$

$= 3 \cdot 6x^2y^3\sqrt{14y}$

$= 18x^2y^3\sqrt{14y}$

10. $(\sqrt{x} - \sqrt{5})^2 =$ $(\sqrt{x} - \sqrt{5})(\sqrt{x} - \sqrt{5})$

$= \sqrt{x^2} - \sqrt{5x} - \sqrt{5x} + \sqrt{25}$

$= x - 2\sqrt{5x} + 5$

11. Simplify: $\dfrac{\sqrt{36x^4}}{\sqrt{18x^7}}$

$= \sqrt{\dfrac{36x^4}{18x^7}}$

$= \sqrt{\dfrac{2}{x^3}}$

$= \dfrac{\sqrt{2}}{\sqrt{x^3}}$

$= \dfrac{\sqrt{2}}{x\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}}$

$= \dfrac{\sqrt{2x}}{x\sqrt{x^2}}$ ($\sqrt{x^2} = x$)

$= \dfrac{\sqrt{2x}}{x^2}$

OR

$\dfrac{\sqrt{36x^4}}{\sqrt{18x^7}} = \dfrac{6x^2}{\sqrt{9 \cdot 2 \cdot x^6 \cdot x}}$

$= \dfrac{6x^2}{3x^3\sqrt{2x}}$

$= \dfrac{2}{x\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}}$

$= \dfrac{2\sqrt{2x}}{2x^2} = \dfrac{\sqrt{2x}}{x^2}$

12. Solve this equation: $6x^2 + 24x = 0$

$6x(x+4) = 0$

$6x = 0$ or $x + 4 = 0$

$x = 0$ or $x = -4$

13. Solve this equation: $\frac{2x}{x-3} - 2 = \frac{1}{x}$

LCD $= x(x-3)$

$$x(x-3)\cdot\left(\frac{2x}{x-3}\right) - x(x-3)\cdot(2) = x(x-3)\cdot\left(\frac{1}{x}\right)$$

$$x(2x) - 2x(x-3) = 1(x-3)$$

$$2x^2 - 2x^2 + 6x = x - 3$$

$$6x = x - 3$$

$$5x = -3$$

$$x = -\frac{3}{5}$$

14. Solve this equation: $\sqrt{16-3x}+2-x=0$

$\sqrt{16-3x}+2-x=0$

$-2+x \quad -2+x$

$\sqrt{16-3x} = x-2$

$*\ (\sqrt{16-3x})^2 = (x-2)^2$

$16-3x = (x-2)(x-2)$

$16-3x = x^2-4x+4$

$-16+3x \quad +3x-16$

$0 = x^2-x-12$

$0 = (x-4)(x+3)$

$x-4=0$ or $x+3=0$

$x=4$ $\quad$ $x=-3$

* A check is required because we squared both sides of our equation.

check: $x=4$

$\sqrt{16-3(4)}+2-(4)=0$

$\sqrt{4}+2-4=0$

$2-2=0$

$0=0$ ✓

check: $x=-3$

$\sqrt{16-3(-3)}+2-(-3)=0$

$\sqrt{16+9}+2+3=0$

$\sqrt{25}+5=0$

$5+5=0$

$10=0$ ✗

$x=4$

Solve each equation:

15. Solve this equation by completing the square:
(If you use any other method, you will not receive credit for this problem.)

$2x^2 = 4x + 10$

$\frac{2x^2}{2} - \frac{4x}{2} = \frac{10}{2}$

$x^2 - 2x + 1 = 5 + 1$

$\frac{1}{2}(-2) = -1$ $\quad (-1)^2 = 1$

$(x-1)^2 = 6$

$\sqrt{(x-1)^2} = \sqrt{6}$

$x - 1 = \pm\sqrt{6}$

$x = 1 \pm \sqrt{6}$

16. Solve using the quadratic formula: $3x^2 + 6x - 2 = 0$
(If you use any other method, you will not receive credit for this problem.)

$a = 3$
$b = 6$
$c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(3)(-2)}}{2(3)}$ $\quad 36 + 24 = 60$

$x = \frac{-6 \pm \sqrt{60}}{6}$

$\sqrt{60} = \sqrt{4 \cdot 15} = 2\sqrt{15}$ (4 · 15)

$x = \frac{-6 \pm 2\sqrt{15}}{6}$

$x = \frac{2'(-3 \pm \sqrt{15})}{6_3}$ $\quad$ so $x = \frac{-3 \pm \sqrt{15}}{3}$

17. Solve this equation using any appropriate method:

$$\frac{6}{x^2-1}-\frac{1}{2}=\frac{1}{x+1}$$

$$\frac{6}{(x+1)(x-1)}-\frac{1}{2}=\frac{1}{x+1}$$

LCD $= 2(x+1)(x-1)$

$$2(x+1)(x-1)\cdot\frac{6}{(x+1)(x-1)}-2(x+1)(x-1)\cdot\frac{1}{2}=2(x+1)(x-1)\cdot\frac{1}{x+1}$$

$$2(6)-(x+1)(x-1)=2(x-1)$$

$$12-(x^2-1)=2x-2$$

$$12-x^2+1=2x-2$$

$$13-x^2=2x-2$$

$$+2 \quad -2x \quad -2x+2$$

$$-x^2-2x+15=0$$

$$\frac{}{-1} \quad \frac{}{-1} \quad \frac{}{-1} \quad \frac{}{-1}$$

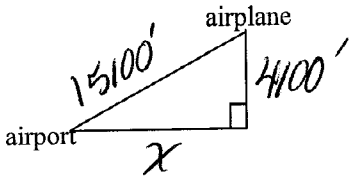
$$x^2+2x-15=0$$

$$(x+5)(x-3)=0$$

$$x+5=0 \quad \text{or} \quad x-3=0$$

$$x=-5 \qquad x=3$$

18. An airport is flying at an altitude of 4,100 feet. The slanted distance directly to the airport is 15,100 feet. How far is the airplane horizontally from the airport. How far is the airplane horizontally from the airport. (Round to the nearest hundredth of a foot).




$a^2+b^2=c^2$

$4100^2+x^2=15100^2$

$x^2=15100^2-4100^2$

$x^2=211{,}200{,}000$

$x=\sqrt{211{,}200{,}000}$

$x\approx 14{,}532.72$

The plane is horizontally 14,532.72 feet from the airport.

19. Solve the system of equations by using the elimination method.

① $-4.5x + 7.5y = 6$

② $-x + 1.5y = 5$

$10$ ① $-45x + 75y = 60$

$10$ ② $-10x + 15y = 50$

① $-45x + 75y = 60$

$-5$ ② $50x - 75y = -250$

$5x = -190$

$x = -38$

into ② $-(-38) + 1.5y = 5$

$38 + 1.5y = 5$

$-38 \quad -38$

$1.5y = -33$

$y = -22$

$(-38, -22)$

OR

① $-4.5x + 7.5y = 6$

$-4.5$ ② $4.5x - 6.75y = -22.5$

$.75y = -16.5$

$\frac{.75y}{.75} = \frac{-16.5}{.75}$

$y = -22$

into ②

$-x + 1.5(-22) = 5$

$-x - 33 = 5$

$-x = 38$

$x = -38$

$(-38, -22)$

*20. Using two variables, write a system of linear equations to answer this question:

A boat on a river travels for 2 hours with a 4-mph current. The return trip against the same current took 6 hours. Find the speed of the boat in still water.

$D = R \cdot T$

$x$ = speed of the boat

$4$ = speed of the current

| | D | R | T |
|---|---|---|---|
| with current | $y$ | $x+4$ | 2 |
| against current | $y$ | $x-4$ | 6 |

① $y = 2(x+4)$

② $y = 6(x-4)$

① → ② $2(x+4) = 6(x-4)$

$2x+8 = 6x-24$

$32 = 4x$

$8 = x$

The speed of the boat in still water is 8 miles per hour.

*Remember, for this word problem, be sure to label the **two** variables, write **two** equations (a system of linear equations) for the situation, solve the equations and write a summary sentence!

BONUS Problems: 3 points each

Reminder: For each word problem, be sure to label the variables, write one or two equations for the situation, solve and write a summary sentence.

B1. The sum of the squares of two consecutive odd positive integers is 74. Find the integers.

$x$ = 1st consec. odd integer

$x+2$ = 2nd " " "

$x^2 + x+2^2 = 74$

$x^2 + x^2 + 4x + 4 = 74$

$\frac{2x^2}{2} + \frac{4x}{2} - \frac{70}{2} = \frac{0}{2}$

$x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$x+7=0$ or $x-5=0$

$x=-7$ $x=5$

not positive! $x+2=7$

The numbers are 5 and 7!

B2. A boat on a river travels for 2 hours with a 4-mph current. The return trip against the same current took 6 hours. Find the speed of the boat in still water.

Teddy has 54 coins totaling $5.10. If he only has nickels and dimes, how many of each coin does he have?

$x$ = # of nickels

$y$ = # of dimes

① $x+y=54$

② $5x+10y=510$

$-5①\ -5x-5y=-270$

$5y=240$

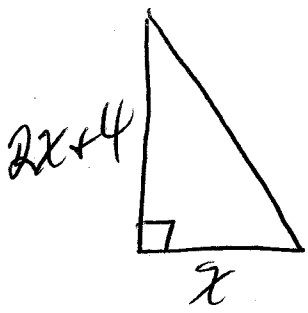
$y=48$

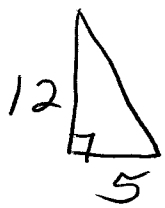
$x+48=54$

$x=6$

Teddy has 6 nickels and 48 dimes.

B3. Two right triangles are similar. The longer leg of the larger triangle is 4 more than twice the shorter leg of that triangle. The longer leg of the smaller triangle is 12, while the shorter leg is 5. How long is each of the legs of the larger triangle?




$\frac{x}{2x+4} = \frac{5}{12}$ or mult by LCD = $12(2x+4)$

$12x = 5(2x+4)$

$12x = 10x + 20$

$2x = 20$

$x = 10$

$2x+4 = 24$

The legs of the larger triangle are 10 units and 24 units long.