

EE 215 Fundamentals of Electrical Engineering

Lecture Notes

Energy Storage Devices (Capacitors and Inductors)
8/7/01

Rich Christie

Overview:

So far, our circuit elements are either sources that supply energy to the circuit from the external world or resistors that dissipate energy from the circuit to the outside world. Now we will look at components that store and release energy. It turns out that these will create circuit equations that are linear differential equations, so you may want to review how to solve such things.

Capacitors:

The *capacitor* stores energy in the electric field. The easy way to make a capacitor is to take two plates of conducting material and separate them with a dielectric (an insulator). The capacitance, C , is then

$$C = \frac{\epsilon A}{d}$$

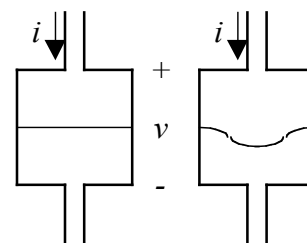
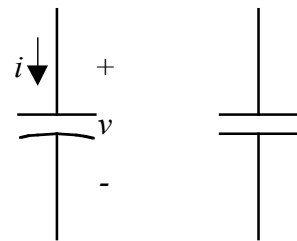
where ϵ is the permittivity of the dielectric (basically how good an insulator it is), A is the area and d is the distance between the plates. This gives rise to the circuit symbol.

Electrically, as current flows through a capacitor, positive charge accumulates on one of the plates and negative charge accumulates on the other plate, so the net charge in the capacitor remains zero.

We can use a water flow analogy to understand the capacitor. It's like a chamber with a rubber membrane stretched across it. Remember that in the water flow analogy, pressure is analogous to voltage. So when charge (water) flows into the capacitor (current flows in), the same amount of water (charge) flows out the other side (current flows through). But the membrane stretches (the electric field becomes stronger between the plates) and so the pressure (voltage) gets higher.

Thus the more charge we stuff in one end, the higher the voltage gets (positive at that end). For capacitors, this is a linear relationship. This gives us the basic relationship

$$q = Cv$$



Where C is the capacitance in Farads (named after Michael Faraday, British physicist). Usual values of C are quite small compared to 1 Farad and come in microfarads, μF , 10^{-6} F, and picofarads, pF, called puffs, $\mu\mu\text{F}$, 10^{-12} F. Think of C as the stiffness of the membrane.

Recalling that current is change in charge with time,

$$i = \frac{dq}{dt} = \frac{d}{dt} C v = C \frac{dv}{dt}$$

gives us the **branch relationship for capacitors**,

$$\boxed{i = C \frac{dv}{dt}}$$

You can think of this as Ohm's Law for capacitors. Note that voltage and current conform to the passive sign convention. Clearly this relationship makes circuit equations into differential equations!

Power in Capacitors:

Recall that power is

$$p = vi$$

and flows from the circuit to the element with voltage and current polarities conforming to the passive sign convention. For the capacitor, power is energy flowing into the capacitor where it is stored. Power is not dissipated to the external world in an ideal capacitor. Then the energy stored in the capacitor is the integral of power:

$$w = \int p dt = \int v i dt = \int v C \frac{dv}{dt} dt = C \int v dv = \frac{1}{2} C v^2 + \text{const}$$

What is this constant, and what are the limits of integration?

Well, the limits are from the birth of the universe (which we can take as $t = -\infty$) to now. The constant is then the energy stored in the capacitor at the birth of the universe, which we will take as zero. So,

$$\boxed{w = \frac{1}{2} C v^2}$$

(One of the things to look for in a capacitor problem is the initial amount of stored energy. Mathematically this corresponds to the initial condition of the capacitor branch relationship.)

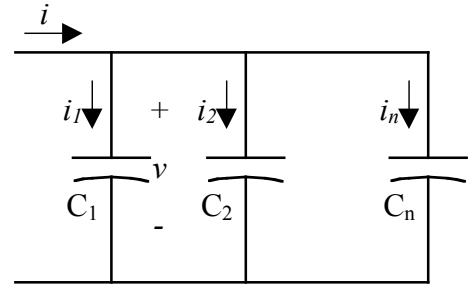
Parallel and Series Capacitors:

Let's take a look at parallel capacitors. They all have the same voltage across them. By KCL,

$$i = i_1 + i_2 + \dots + i_n$$

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \dots + C_n \frac{dv}{dt}$$

$$i = (C_1 + C_2 + \dots + C_n) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$



Pretty clearly, *parallel capacitors add*.

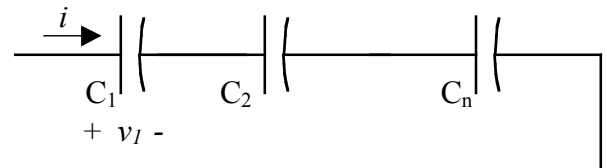
$$C_{eq} = C_1 + C_2 + \dots + C_n$$

Series capacitors, however...

$$v = v_1 + v_2 + \dots + v_n$$

$$v = \frac{1}{C_1} \int idt + \frac{1}{C_2} \int idt + \dots + \frac{1}{C_n} \int idt$$

$$v = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} \right) \int idt = \frac{1}{C_{eq}} \int idt$$



or

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

which looks a lot like the parallel resistor formula. In fact

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

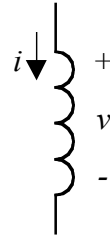
Inductors:

If energy can be stored in an electric field, it can also be stored in a magnetic field. This is done by **inductors**. Inductors are made by making coils of wire. When current flows in the wire, it creates a magnetic field that stores energy. Inductance depends on geometry. For a coil of N turns, cross sectional area A , and length l , inductance L in henries is

$$L = \frac{\mu N^2 A}{l}$$

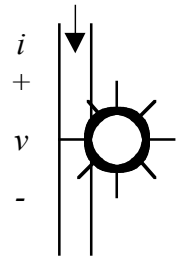
Here μ is the permeability of the core material, a fundamental physical constant. L is in Henries, named for Joseph Henry, a US physicist. Usual values are in mH, 10^{-3} H.

The coil gives the circuit symbol for inductors:



The water flow analogy for an inductor involves a water wheel. When pressure (voltage) is applied across the wheel (inductor), it starts to accelerate, moving more and more water (current). The momentum of the wheel represents the energy stored in the magnetic field by the flowing current.

From this we can see that the voltage causes the current to increase over time - to change. The inductance is the coefficient of change, sort of the rotational inertia of the wheel.



$$v = L \frac{di}{dt}$$

is the *inductor branch relationship* (Ohm's Law equivalent). Again, it is in passive sign convention. Then

$$p = vi$$

$$w = \int v i dt = \int L \frac{di}{dt} i dt = L \int i di = \frac{1}{2} L i^2$$

$$w = \frac{1}{2} L i^2 \text{ is the stored energy in an inductor, in joules.}$$

Series inductors have the same current

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots + L_n \frac{di}{dt} = (L_1 + L_2 + \dots + L_n) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \dots + L_n$$

Series inductors add like series resistors.

Parallel inductors combine like parallel resistors.

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \quad L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}}$$