EE 215A Fundamentals of Electrical Engineering
Lecture Notes

Operational Amplifiers (Op Amps)
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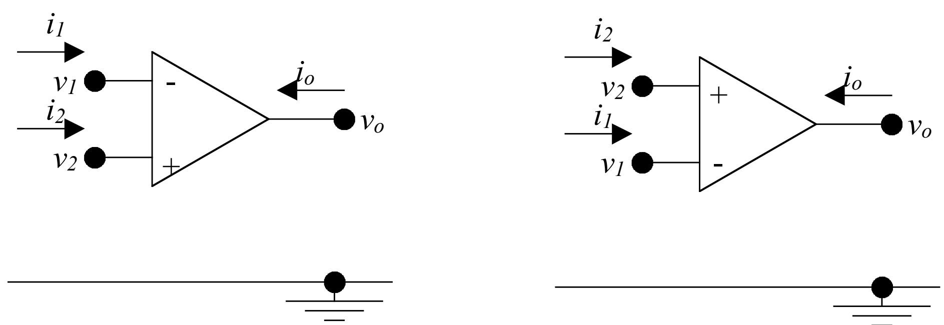
Overview:

The operational amplifier, or op amp for short, is a fundamental building block in circuit design. Stuffed inside a chip are a bunch of transistors and other elements that make up a near-ideal voltage controlled voltage source (VCVS) with near-infinite gain. We'll start by assuming it is ideal.

Circuit Symbol:

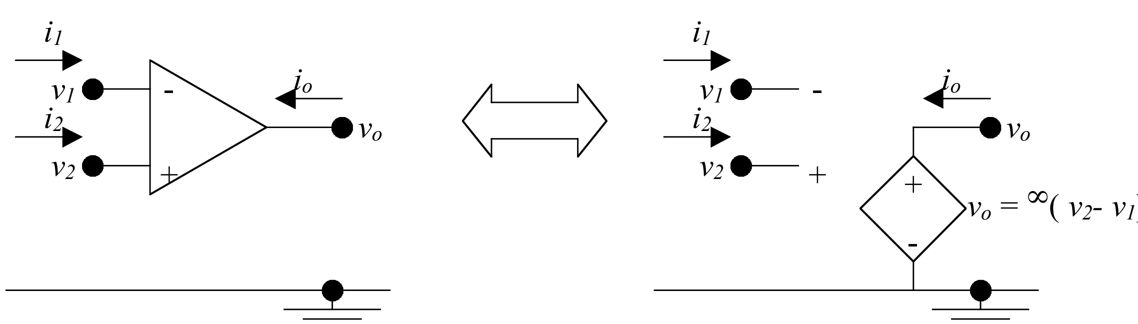
The op amp has a special circuit symbol, a triangle with two inputs and one output



The + and - connections are the non-inverting input and the inverting input. These are usually, *but not always,* drawn with the minus (-) at the top.

Ideal Op Amps Relationships:

An ideal op amp is an ideal voltage controlled voltage source. We can think of the op amp symbol being replaced as follows:



Notice what this implies:

If v_o is finite, then

$$v_2 - v_1 = \frac{v_o}{\infty} = 0$$

$$\boxed{v_1 = v_2}$$

and

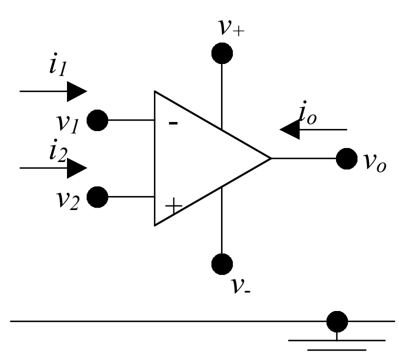
$$\boxed{\begin{array}{l} i_1 = 0 \\ i_2 = 0 \end{array}}$$

These are in fact the *ideal op amp relationships*, and once we get used to them we can use them to solve op amp circuits.

(At this point you might be wondering why the op amp has its own special symbol when it's really just a VCVS. The answer has to do with packaging. The op amp comes in a single package which includes the inverting and non-inverting inputs. The op amp symbol indicates that the package combines all the connections in one place, where the VCVS symbol lets the inputs be anywhere.)

Op Amp Power Supplies:

But wait, if the input currents i_1 and i_2 are zero, where does the output current i_o come from? The dependent voltage source, in the model, but in real life, we have to supply the op amp with an external power supply. Many real world op amps (the LM741, for example) require both positive and negative voltage (as opposed to positive and ground, called single source.)

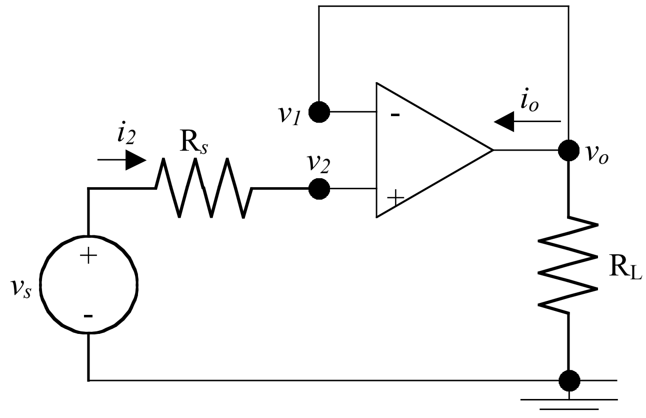


If you really want to do KCL for the op amp (for a supernode containing the op amp, or for the output node), you have to include the power supply. Usually it's not necessary and the power supply connections are omitted - we all understand that they are there, so why clutter up the circuit with them?

Op Amp Circuit Analysis:

Let's try a circuit with an op amp.

What's the relationship between v_s and v_o?



$v_o = v_1$ since they are connected.
$v_2 = v_1 = v_o$ from the ideal op amp relationships. Since $i_2 = 0$, $v_2 = v_s$ and thus
$v_o = v_s$

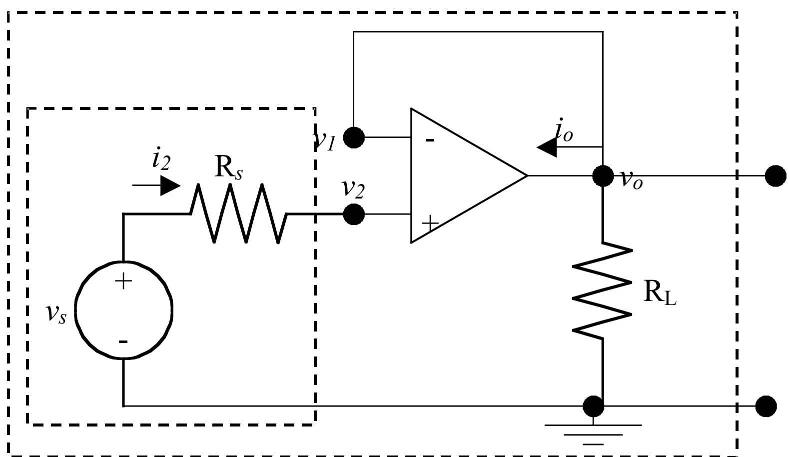
This circuit is called a *voltage follower* or a buffer amplifier.

At first glance we have not done much. But let's ask: consider the source (connected to v_2) before it is connected to the op amp. What's the Thevenin equivalent resistance? It's pretty obvious that

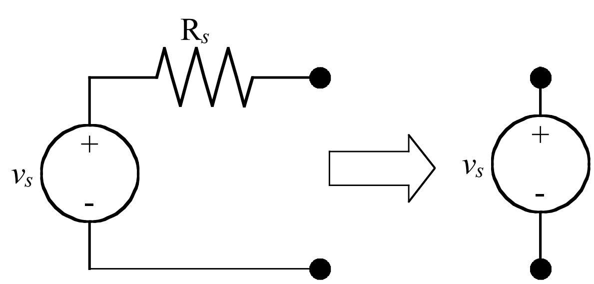
$$R_t = R_s$$

Now consider the Thevenin equivalent resistance seen looking in to the whole thing. v_{oc} is just v_o, and i_{sc} is - well, it's infinite!

(v_2 must be v_s because i_2 is zero, then v_1 must equal v_2 which means v_o is v_s no matter what the current is but it is shorted -> infinite current.)



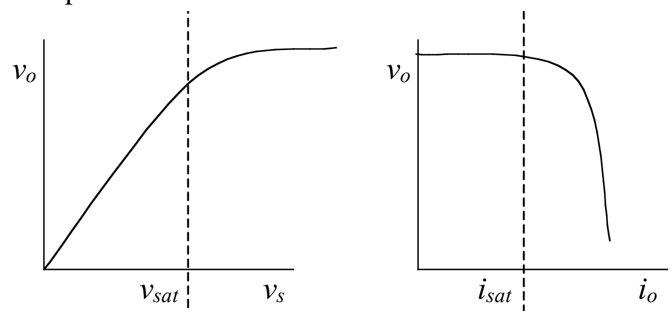
So $R_t = \frac{v_{oc}}{i_{sc}} = 0$ and the Thevenin equivalent is an ideal voltage source of value v_s:



Saturation:

If we actually built one of these voltage followers and varied the input voltage v_s, we would see an interesting relationship between input and output.

If we kept v_s constant, and varied R_L so the output current varied, we would also see something unexpected.



What's happening is that the op amp is *saturating*, that is, moving out of the linear region and ceasing to operate as an amplifier. We can only model the op amp as ideal when it is not saturated.

We can check saturation, though:

$$|v_o| \le v_{sat}$$
$$|i_o| \le i_{sat}$$

v_{sat} and i_{sat} come from the specification sheet. v_{sat} is usually a volt or two less than the power supply voltage. For example, if the power supply is +/- 12V and i_{sat} is 2mA, then the maximum output voltage is 11 V

If v_{sat} is 10V, what's the maximum R_L? (infinite, haha. Maybe a more interesting question is) What's the minimum?

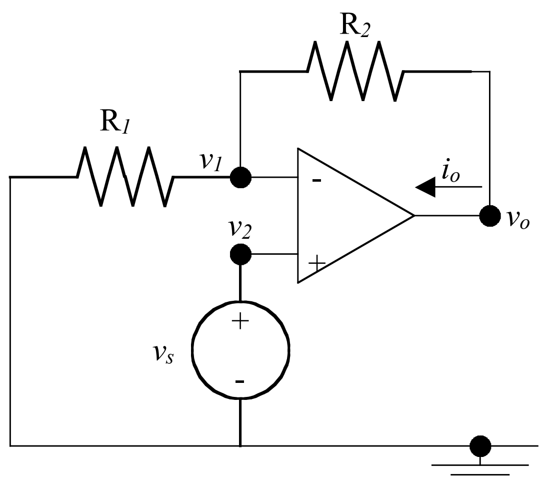
$$i_o \le i_{sat}$$
$$\frac{v_o}{R_L} \le 2mA$$
$$\frac{10}{R_L} \le 2mA$$
$$R_L \ge \frac{10}{2 \times 10^{-3}} = 5k\Omega$$

Slew Rate:

At this point we should probably note that another departure from ideality in the op amp is that the slew rate, $\frac{dv_o}{dt}$, is also limited. This affects how fast the op amp responds to a step change in input, and how high a frequency it can operate at. We won't pay much attention to slew rate in this course, but you will run into it in later courses, and in the real world!

More Analysis:

Let's look at another circuit (you can do the analysis)



As before, find v_o as a function of v_s.

First, what's v_2? v_s.

So what's v_1? v_s

Now write KCL at v_1:

$$\frac{v_s}{R_1} + \frac{v_s - v_o}{R_2} = 0$$

$$v_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{v_o}{R_2} = 0$$

$$v_o = v_s\left(1 + \frac{R_2}{R_1}\right)$$

This is called a *non-inverting amplifier*. Notice that the gain of the amplifier can be changed by changing the ratio of the resistors, for example with a potentiometer for R_2.

Design with Op Amps:

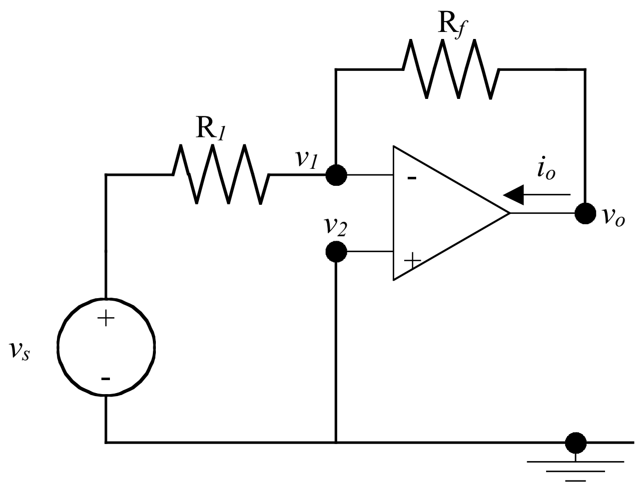
Somewhere in your textbook (or some textbook somewhere!) is a catalog of op amp circuits that perform different functions. There are lots beyond what's in the book. The circuits are generic, that is, they do not have specific values for the resistances. The catalog is a starting point for circuit design. We now know enough to do some almost real world design. The process is:

- Given a function to perform
- Choose a circuit from the catalog that performs the function. (Or invent a new one. If you do, patent it.)
- Figure out values for the circuit components so that the specific function we want is implemented (e.g. specific gain).
- Check the circuit against limits.

- Iterate as required.

Example: We want to amplify an input signal by a factor of 100 (gain of 100). The input signal has a range of 0 to -100 mV, but we want a positive voltage output.

Pretty clearly we want the inverting amplifier circuit from the catalog:



Let's check the analysis:

$$v_2 = 0$$
$$v_1 = 0$$
$$\frac{v_s - 0}{R_1} + \frac{v_o - 0}{R_f} = 0$$
$$v_o = -\frac{R_f}{R_1} v_s$$

For our design we want

$\frac{R_f}{R_1} = 100$ but that leaves a lot of choice. What other concerns are there?

Source current: Suppose the source is limited to 10 mA output current. Then

$$\frac{v_s}{R_a} < 10mA$$
$$\frac{100mV}{10mA} < R_1$$
$$R_1 > 10\Omega$$
$$R_f = 1K\Omega$$

Is this a good design?

What about v_{sat}?

$$v_{o\max} = -100 \cdot (-100mV) = 10V$$

We'll need a power supply of around +/- 11V. +/- 12 V is a common standard (you can find this voltage inside many desktop PCs) or +/- 15V, both are OK but NOT +/- 5V, which is the most common digital electronic power supply voltage, although lower voltages (3.3 V, 1.6 V) are gaining.

What about i_{sat}?

$$i_o = \frac{v_o}{R_f} = \frac{10}{1K} = 10mA(!)$$

Whoops, the limit we talked about earlier was 2 mA. (This limit will vary from op amp type to op amp type, so maybe we just need to buy a different op amp. But let's use the one in the parts kit… with a 2mA output current limit.)

If we increase R_f, and increase R_1 to keep the 100 ratio, then i_s will also go down, and we will meet both constraints.

$$i_o < 2mA$$

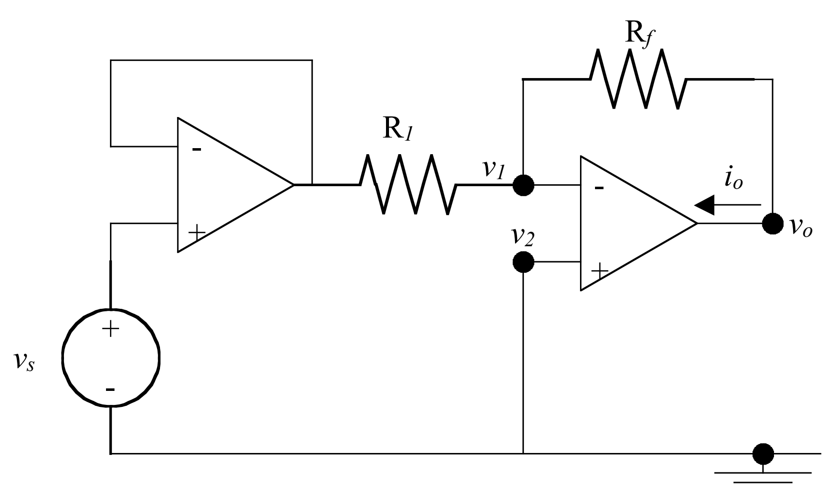
$$\frac{v_o}{R_f} < 2mA$$

$$R_f > \frac{10}{2mA} = 5k\Omega$$

$$R_1 = \frac{Rf}{100} = 50\Omega$$

Note this also meets the general guideline to use 5 kΩ – 100 kΩ resistors around an op amp.

We should probably also worry about resistor power dissipation, but will leave that up to you.



What could we do if the source current limit was much, much smaller?

We could cascade op amps, using a voltage follower then inverting amp, for example.

Real Op Amps:

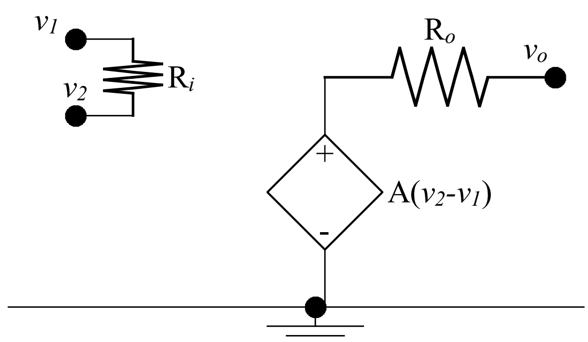
The ideal op amp has already been brought closer to reality by adding saturation and slew rate limits, but the core component was still an ideal VCVS.

In real op amps the VCVS is not ideal. In particular, real op amps have:

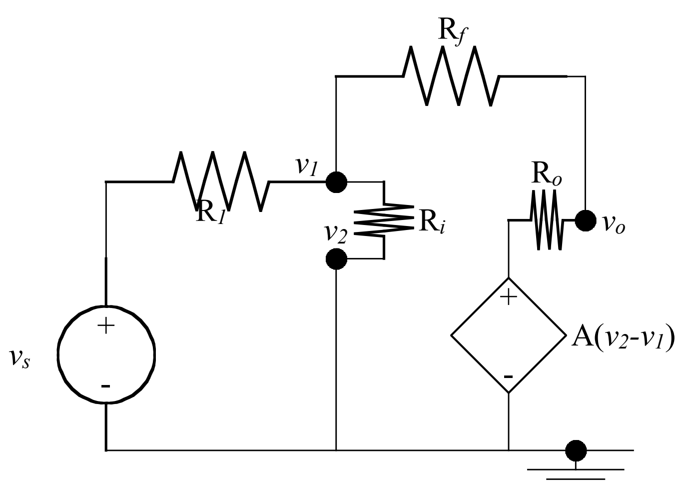
a finite (but large) gain, 200,000 versus infinite in the ideal model
a finite (but large) input resistance, 2 MΩ versus infinite
a small output resistance, 75 Ω versus zero

Tiny bias currents, 80 nA versus zero
Small offset voltage, 1 mV versus zero

In many, dare I say most cases, these imperfections are simply ignored and the ideal op amp analysis (with saturation limits) is good enough for most work. It's certainly good for thinking up new op amp circuits. However, there are applications where the imperfections affect performance. They are usually accounted for with more detailed models. For example, the finite gain and resistance model is:



Let's apply this model to the inverting amplifier we deisgned and see how much error we get.



We want to get the relationship between v_s and v_o and compare it to the ideal version. Let's write KCL at v_1:

$$\frac{v_1 - v_s}{R_1} + \frac{v_1}{R_i} + \frac{v_1 - v_o}{R_f} = 0$$

and at v_o,

$$\frac{v_o - v_1}{R_f} + \frac{v_o - (A(-v_1))}{R_o} = 0$$

From the second equation

$$R_o(v_o - v_1) + R_f(v_o + Av_1) = 0$$

$$v_o(R_o + R_f) = v_1(R_o - AR_f)$$

$$v_1 = \frac{R_o + R_f}{R_o - AR_f} v_o$$

Subbing in to the first equation

$$\frac{v_1 - v_s}{R_1} + \frac{v_1}{R_i} + \frac{v_1 - v_o}{R_f} = 0$$

$$v_1(R_iR_f + R_1R_f + R_iR_1) - v_sR_iR_f - v_oR_1R_i = 0$$

$$v_o\frac{R_o + R_f}{R_o - AR_f}(R_iR_f + R_1R_f + R_iR_1) - v_sR_iR_f - v_oR_1R_i = 0$$

$$v_o\left[\frac{(R_o + R_f)(R_iR_f + R_1R_f + R_iR_1) - R_1R_i(R_o - AR_f)}{R_o - AR_f}\right] = v_sR_iR_f$$

$$v_o = \frac{R_iR_f(R_o - AR_f)}{R_oR_iR_f + R_oR_1R_f + R_oR_iR_1 + R_iR_f^2 + R_1R_f^2 + R_iR_1R_f - R_1R_iR_o + AR_1R_iR_f} v_s$$

$$v_o = \frac{R_iR_f(R_o - AR_f)}{R_oR_iR_f + R_oR_1R_f + R_iR_f^2 + R_1R_f^2 + R_iR_1R_f + AR_1R_iR_f} v_s$$

$$v_o = \frac{R_iR_f(R_o - AR_f)}{R_f(R_oR_i + R_oR_1 + R_iR_f + R_1R_f + R_iR_1 + AR_1R_i)} v_s$$

$$v_o = \frac{R_i(R_o - AR_f)}{R_o(R_i + R_1) + R_f(R_i + R_1) + R_iR_1(1 + A)} v_s$$

$$v_o = \frac{R_i(R_o - AR_f)}{(R_o + R_f)(R_i + R_1) + R_iR_1(1 + A)} v_s$$

Whew!

First let's see if this reduces to the ideal relationship with $R_o = 0$, $R_i = \infty$ and $A = \infty$.

$$v_o = \frac{-AR_iR_f}{R_f(R_i + R_1) + R_iR_1(1+A)}v_s$$

$$v_o = \frac{-AR_f}{\frac{R_fR_i}{R_i} + \frac{R_fR_1}{R_i} + R_1(1+A)}v_s = \frac{-AR_f}{R_f + R_1(1+A)}v_s$$

$$v_o = \frac{-R_f}{\frac{R_f}{A} + \frac{R_1}{A} + R_1\frac{A}{A}}v_s = -\frac{R_f}{R_1}v_s$$

And this matches.

Now for some numbers. In our ideal design R_1 = 50Ω, R_f = 5 kΩ and $v_o = -100v_s$.

In the more detailed circuit, A = 200,000, R_i = 2 MΩ, R_o = 75 Ω (LM 741 numbers)

$$v_o = \frac{R_i(R_o - AR_f)}{(R_o + R_f)(R_i + R_1) + R_iR_1(1+A)}v_s$$

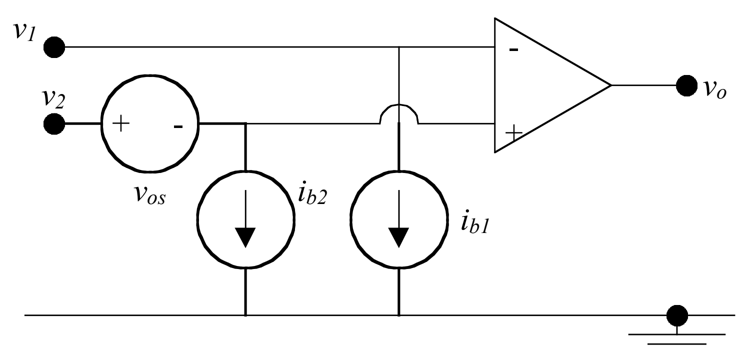
$$v_o = \frac{2\times 10^6(75 - 200{,}000\cdot 5{,}000)}{(75+5000)(2\times 10^6 + 50) + 2\times 10^6\cdot 50\cdot 200{,}001}v_s$$

$$v_0 = -99.9488v_s$$

As we can see it's not a major accuracy hit, about 0.05%.

Bias Currents and Offset Voltages:

The other way that op amps deviate from ideality is the presence of bias currents and offset voltages. Bias currents are due to asymmetries in the op amp design and bias voltages to the inherent electrophysics. They just result in more complex models:



Of course, these additions can be combined with the finite gain and internal resistance model if desired.

For a voltage follower, the typical effect of bias and offset is one thousandth of one percent. Thus they don't get a lot of attention!

Common Mode Gain:

Even in the finite gain model of the op amp, the gain is considered constant. In fact, it depends on the average of the input voltages and also on the frequency of the input.

The dependence on average is called *common mode gain*. The VCVS voltage equation is really

$$A(v_2 - v_1) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

The common mode gain A_{cm} is hidden in the specs as the common mode rejection ratio (CMRR). (No engineer social life jokes, please).

$$CMRR = \frac{A_{cm}}{A}$$

With CMRRs of 31,600 (as the LOW value) this can often be ignored.

Variation of gain with frequency is similarly disguised in the specs as gain-bandwidth product. What really stays constant is gain times frequency (dc and near-dc is a special case!)

$$B = A(f) \cdot f$$

$$A(f) = \frac{B}{f}$$

And that's pretty much it for op amps until we know something about energy storage devices.