

EE 215 Fundamentals of Electrical Engineering

Lecture Notes

Superposition

8/2/01

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Superposition:

Overview:

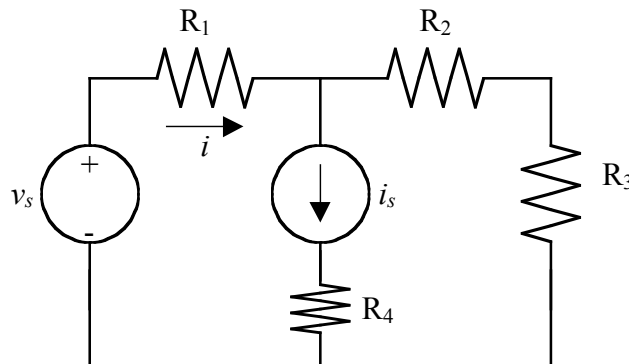
The basic idea behind superposition is that you can take a circuit with several **independent** sources (voltage or current), and find a particular circuit value (voltage or current) by adding the effects of each source considered separately.

It's a powerful opportunistic circuit solution method that can transform a difficult solution into a series of simple ones.

This only works if all the circuit elements are linear, i.e. constant resistance, and linear expressions for dependent sources.

It only works for voltage and current, NOT for power.

i.e. suppose we want to find current i for this circuit:



Suppose we could find i_v , which is current i with just the voltage source, and i_i , which is current i with just the current source. Then superposition says

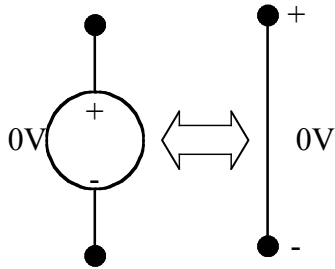
$$i = i_v + i_i$$

But what does "just the voltage source" mean?

It means setting the value of

the other **independent** sources to zero.

Consider a voltage source with a value of zero. Current can flow through it. What else can current flow through that has a voltage difference of zero?



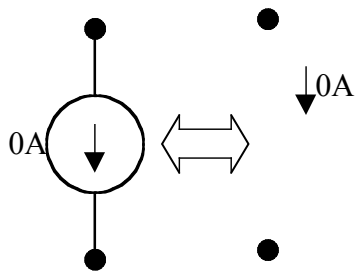
Your choices are really a short circuit or an open circuit, and it should be clear that

A voltage source with zero voltage is a short circuit.

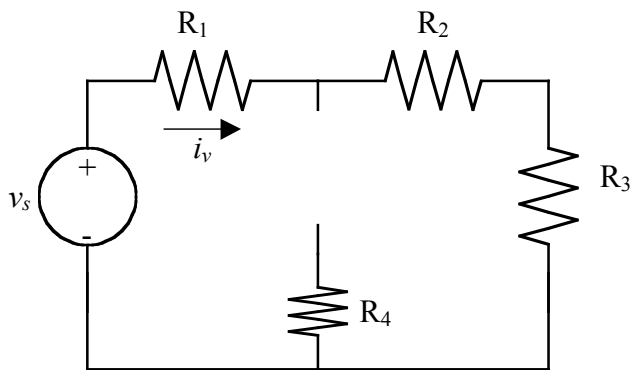
"Zeroing out" a voltage source means replacing it with a short circuit.

For current sources, well, open circuits have zero current, so

"Zeroing out" a current source means replacing it with an open circuit.



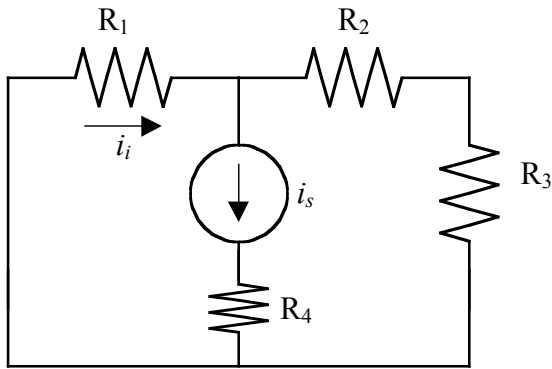
Let's apply this to the example circuit: zeroing out the current source,



Note how the open circuit simplifies the rest of the circuit. It's just Ohm's Law and series resistors.

$$i_v = \frac{v_s}{R_1 + R_2 + R_3}$$

Now let's zero out the voltage source.



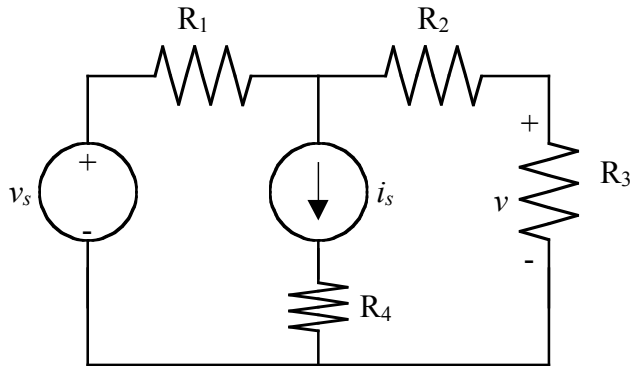
Again, the circuit is simplified. We recognize the current divider:

$$i_i = i_s \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

And now just add them up:

$$i = i_v + i_i = \frac{v_s}{R_1 + R_2 + R_3} + i_s \frac{R_2 + R_3}{R_1 + R_2 + R_3} = \frac{v_s + i_s(R_2 + R_3)}{R_1 + R_2 + R_3}$$

The real secret is to be able to do this without redrawing the circuit each time. I tend to hold my thumb over the zeroed source while writing equations. A good circuit theory geek can write the equation for i straight off. Let me try it for v :

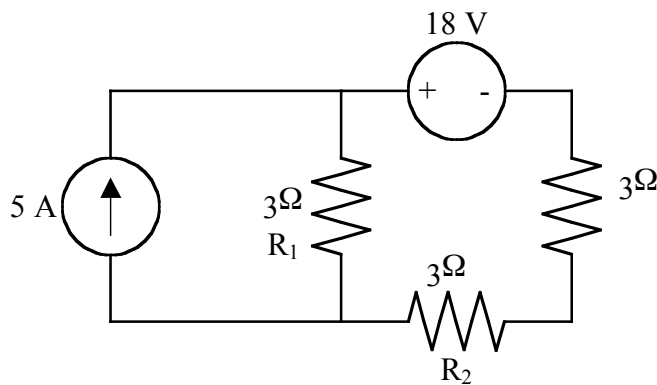


$$v = v_s \frac{R_3}{R_1 + R_2 + R_3} - R_3 i_s \frac{R_1}{R_1 + R_2 + R_3} = \frac{R_3(v_s - i_s R_1)}{R_1 + R_2 + R_3}$$

In this case the first term is just voltage divider. The second term is a current divider, with Ohm's Law (the R_3 in front) and some attention to polarity, so it is a three-in-one! But each of the three laws is simple.

Remember, this only works for voltage and current, NOT power:

$$p = i^2 R_1 = (i_v + i_i)^2 R_1 \neq i_v^2 R_1 + i_i^2 R_1 \text{ (Pythagoras strikes again!)}$$



Let's try: Use superposition to find the power dissipated in the 3 Ω resistor marked R₁:

Let's start by asking you to outline your approach.

Let's agree on finding either current through or voltage across the resistor.

OK current: $i = 5 \frac{6}{9} + \frac{18}{9} = \frac{48}{9} = \frac{16}{3} A$ (i flows down through R₁)

(Did a current divider and then Ohm's Law)

$$p = i^2 R = \left(\frac{16}{3}\right)^2 3 = \frac{256}{9} 3 = 85.3W \text{ (pretty hot!)}$$

OK voltage: $v = 18 \frac{3}{9} + 3 \cdot 5 \cdot \frac{6}{9} = 6 + 10 = 16V$

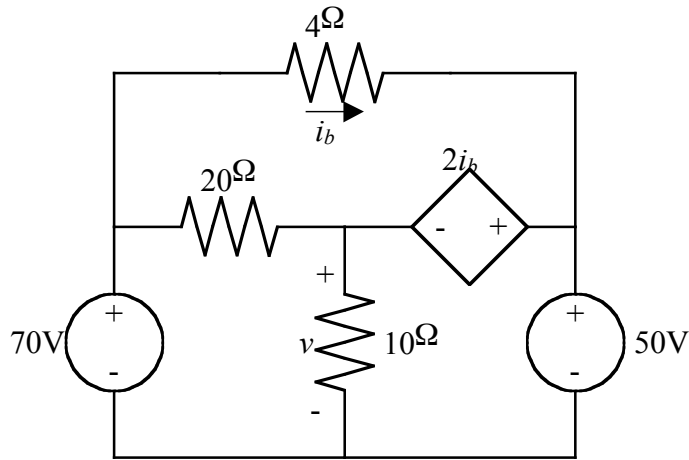
Did a voltage divider and Ohm's Law times current divider.

$$p = \frac{v^2}{R} = \frac{(16)^2}{3} = \frac{256}{3} = 85.3W$$

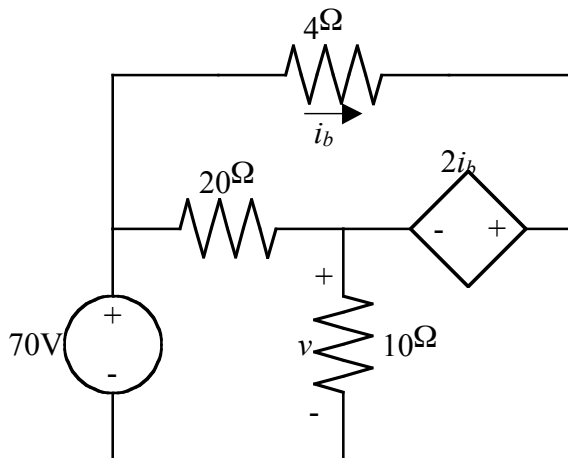
Why not try it for R₂?

Dependent Sources and Superposition:

For dependent sources: Can't zero dependent sources - have to leave in circuit. Tends to make analysis more difficult, still can be a win over nodal or mesh analysis.



Find v .

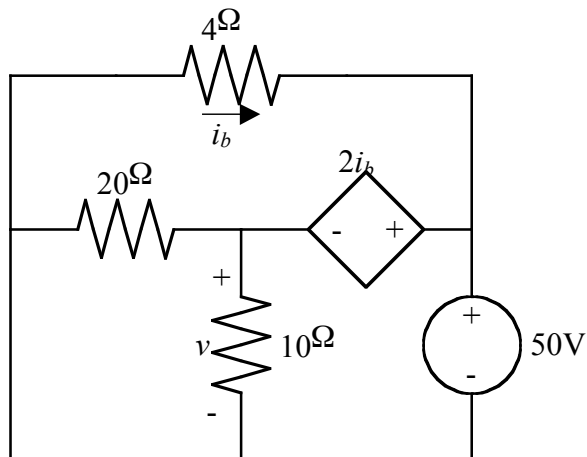


Zero 50 V source, get new ckt.

$$i_b = \frac{70}{4} = 17.5A$$

$$2i_b = 35V$$

$$v = -35V$$



Now zero 70V source.

$$i_b = -\frac{50}{4} = -12.5A$$

$$2i_b = -25V$$

$$v = 50 - (-25) = 75V$$

$$v_{tot} = -35 + 75 = 40V$$