(EE 215 Fundamentals of Electrical Engineering
Lecture Notes

Mesh Analysis
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Mesh Analysis:

Overview:

(The other) systematic method of circuit analysis.
Works on all circuits.
No creative thinking needed.
Computers don't like as much as nodal analysis.
Once over lightly.

Process:

Write KVL for every mesh in terms of *mesh currents*.
Special handling for current sources.

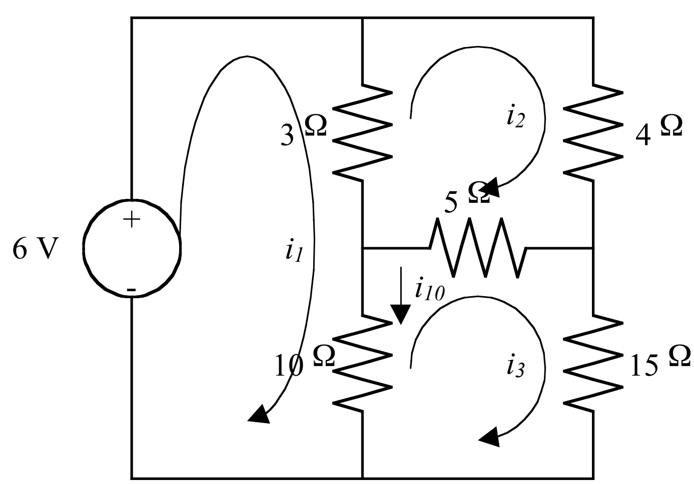
What's a *mesh*? A loop that does not contain any other loops within it.

In essence, every "window" in a planar circuit is a mesh.

Mesh current is current that flows in the mesh.

Branches have either one or two mesh currents flowing through them. If two, they usually flow in opposite directions.

How many meshes in the example? (Three.)



For some reason mesh currents seem to flow clockwise. (Probably CCW below the equator, ba-dump-bump!) Actually this choice is arbitrary. It's useful to have them flow the same way when checking the equations for consistency.

Note that the total current (or, actual current) in a branch is the difference of the branch currents flowing through it. Thus, the current flowing down in the 10Ω resistor is

$$i_{10} = i_1 - i_3$$

(Just like branch voltages were the difference in node voltages.)

Of course, the current flowing down in the 15 Ω resistor is just

$$i_{15} = i_3$$

(Sort of like the branch voltage of a branch connected to the reference node.)

Now write KVL around the loop defined by each mesh, in terms of mesh currents.

Remember how in nodal analysis we used current leaving a node, even though KCL is stated in terms of current entering a node? Well, now we'll use the polarity at the near end of a branch for the sign of the voltage - and for much the same reason.

$$\begin{array}{ll} \text{loop 1} & -6 + 3(i_1 - i_2) + 10(i_1 - i_3) = 0 \\ \text{loop 2} & 3(i_2 - i_1) + 4(i_2) + 5(i_2 - i_3) = 0 \\ \text{loop 3} & 10(i_3 - i_1) + 5(i_3 - i_2) + 15i_3 = 0 \end{array}$$

Note that the mesh current in the loop for which the equation is being written has a positive sign in every term where it appears! This is a good consistency check, and is due to having all mesh currents flow the same way, and taking the near end for polarity.

Solve in the usual way.

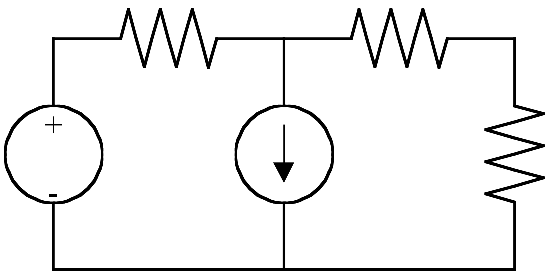
$$\begin{array}{l} 13i_1 - 3i_2 - 10i_3 = 6 \\ -3i_1 + 12i_2 - 5i_3 = 0 \\ -10i_1 - 5i_2 + 30i_3 = 0 \end{array}$$

$i_1 = 0.777$ A
$i_2 = 0.325$ A
$i_3 = 0.313$ A

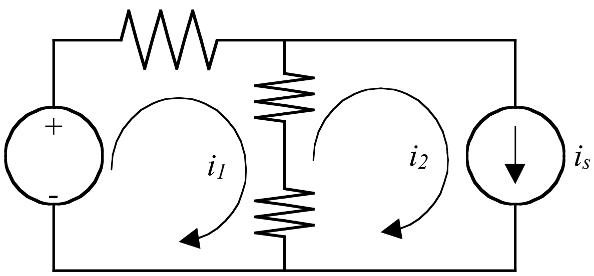
$i_{10} = i_1 - i_3 = 0.464$ A

Current Sources:

First, try to get the current source to where only one mesh current flows through it. Redraw the circuit if you have to. For example:

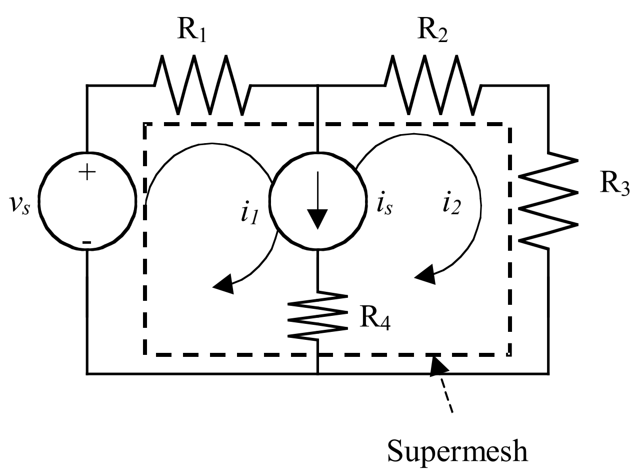


Redraw to:



Now the mesh current is just the source current: $i_2 = i_s$ and write KVL around the other loop.

Sometimes you can't get the current source on the outside. Then you define a *supermesh* which incorporates the meshes on either side of the current source. You write KVL around the supermesh (but in terms of the existing mesh currents), and you get a mesh current relationship (auxiliary equation) from the source.



(Note this example could actually be redrawn if we wanted to…)

From the source:

$$i_s = i_1 - i_2$$

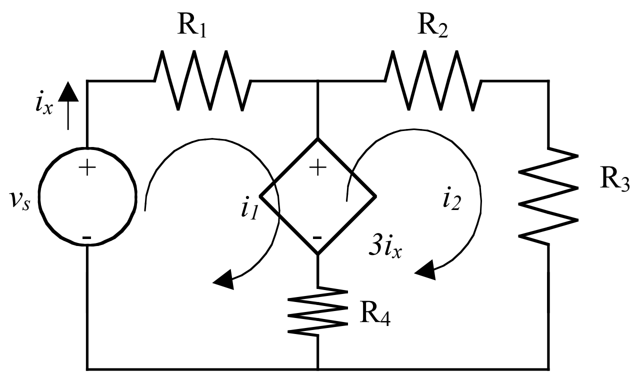
Around the supermesh

$$-v_s + i_1 R_1 + i_2 R_2 + i_2 R_3 = 0$$

And there are two equations in two unknowns.

Dependent Sources:

Just express the source value in terms of mesh currents.



Pretty clearly,

$$3i_x = 3i_1$$

and KVL around loop 1 is

$$-v_s + R_1 i_1 - 3i_1 + R_4(i_1 - i_2) = 0$$

etc.