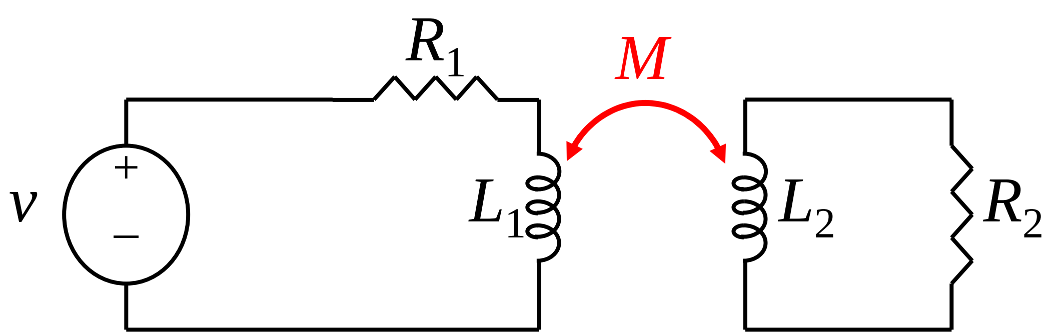
Lecture 14

- **Mutual Inductance**
- **Chapter 6.4-6.5**

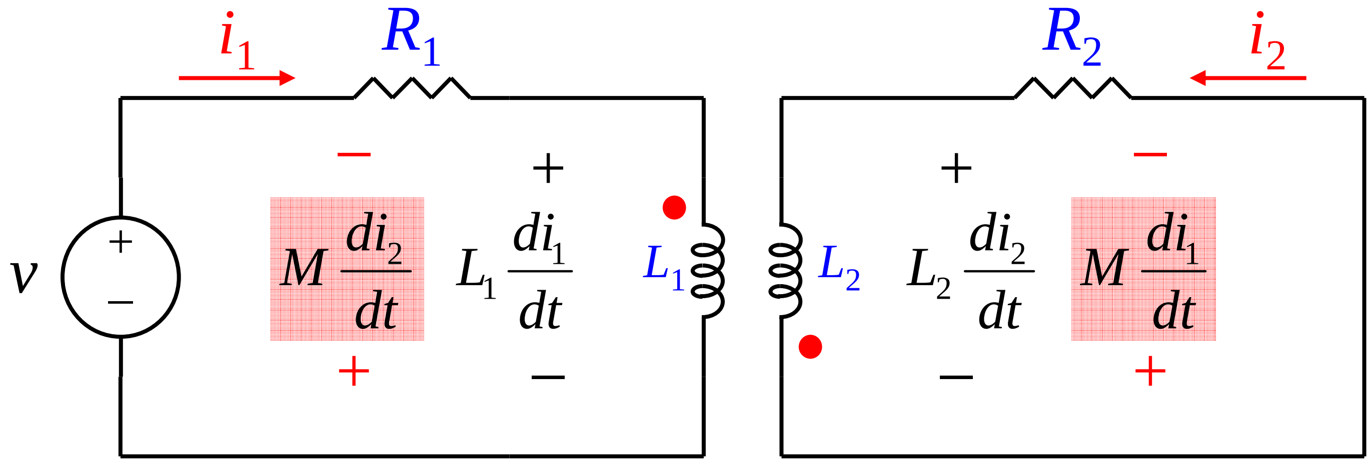
Mutual Inductance

- Def: Inductance is the circuit parameter that relates a voltage to a time-varying current in the same circuit ("self-inductance").
- If a magnetic field links two circuits then we obtain mutual inductance (in addition to self-inductance).



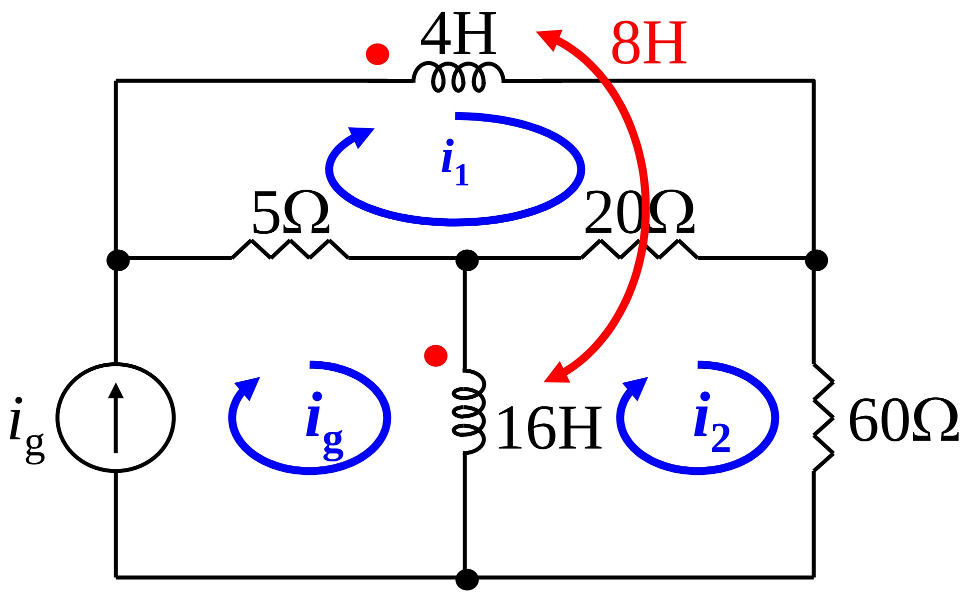
$L_1\, di_1/dt$	$L_2\, di_2/dt$	self-inductance
$+ M\, di_2/dt$	$+ M\, di_1/dt$	mutual inductance

Dot Convention



- When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.
- $-v + i_1 R_1 + L_1\, di_1/dt - M\, di_2/dt = 0$
- $i_2 R_2 + L_2\, di_2/dt - M\, di_1/dt = 0$

Example



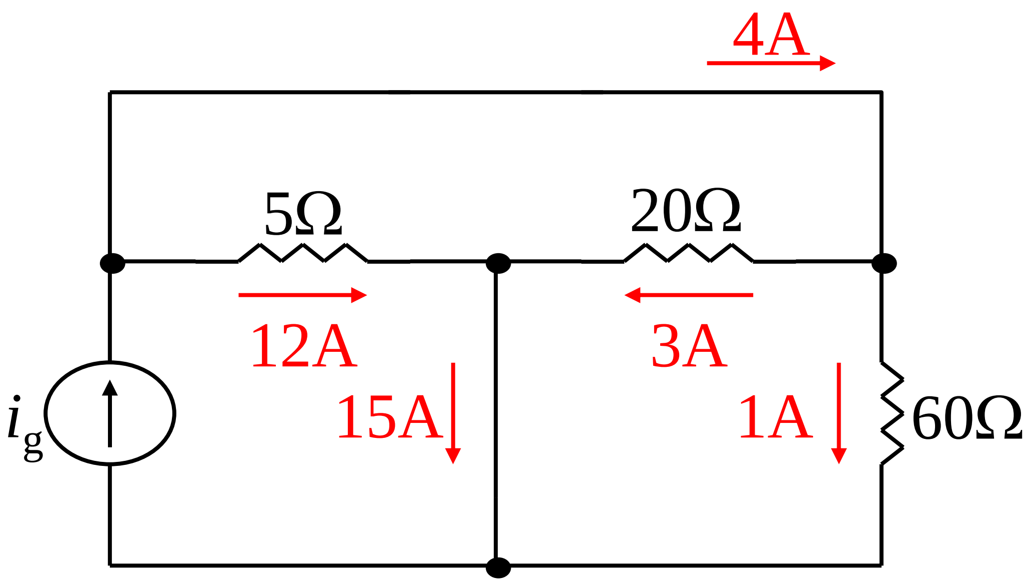
- Mesh current method:

$$4di_1/dt + 8d(i_g – i_2)/dt + 20(i_1 – i_2) + 5(i_1 – i_g) = 0$$

$$20(i_2 – i_1) + 60i_2 + 16d(i_2 – i_g)/dt – 8di_1/dt = 0$$

- If i_g=16A after steady state is reached, then how much energy is stored in this circuit?

Example (in steady state)



- Determine mesh currents:
  - If all mesh currents are constant, then inductors behave like short circuits.

- $1/R_{eq} = 1/5 + 1/20 + 1/60 = 16/60 = 4/15\ \Omega^{-1}$
- $i_g = 16\text{A},\ v_g = 60\text{V},\ i_2 = 1\text{A},\ i_1 = 4\text{A}$
- $W = ½\, L_1\, i_1^2 + ½\, L_2\, (i_g - i_2)^2 + M\, i_1 (i_g - i_2)$

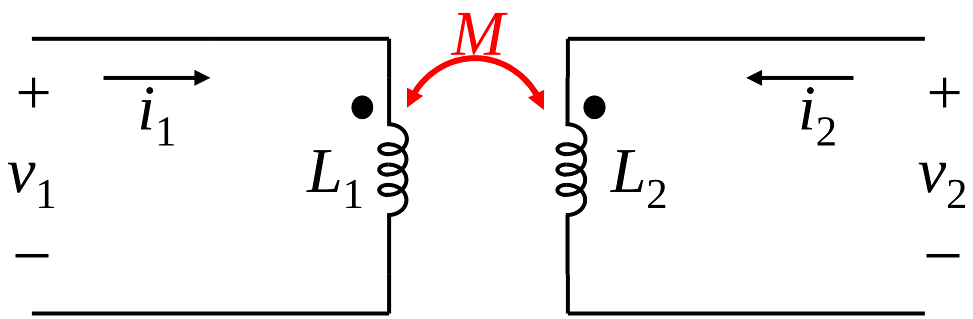
  $= ½\ 4\ 16 + ½\ 16\ 15^2 + 8\ 4\ 15 = 32 + 5000 + 480 = 5512\text{J}$

Coupling Coefficient

- It can be shown that $M^2 = k^2 L_1 L_2$ for a constant k with $0 \leq k \leq 1$.
- k depends on the physical arrangement of the inductors.
  - k = 0 no coupling
  - k = 1 ideal coupling

Energy and Mutual Inductance (1)

- Generic Example:



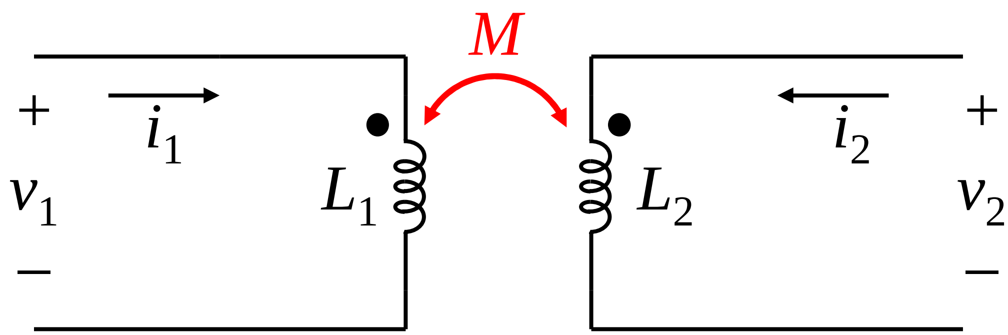
  - Assume that initially, $i_1 = 0$ and $i_2 = 0$.
  - Then gradually increase i_1 to I_1:

$$w_1 = \int_0^{W_1} dw = L_1 \int_0^{I_1} i\,di = \frac{1}{2} L_1 I_1^2$$

  - Then gradually increase i_2 to I_2:

$$w_2 = \int_{W_1}^{W_2} dw = M \int_0^{I_2} I_1 di + L_2 \int_0^{I_2} i\,di = I_1 I_2 M + \frac{1}{2} L_2 I_2^2$$

Energy and Mutual Inductance (2)



sign depends on dots

– Total energy:

$$w = w_1 + w_2 = \frac{1}{2} L_1 I_1^2 + M I_1 I_2 + \frac{1}{2} L_2 I_2^2$$

– This equation holds in general, for any point in time:

$$w = \frac{1}{2} L_1 i_1^2 \pm M i_1 i_2 + \frac{1}{2} L_2 i_2^2$$

– Note that because of $M = k\sqrt{L_1 L_2}$ with $0 \le k \le 1$ w is always non-negative.