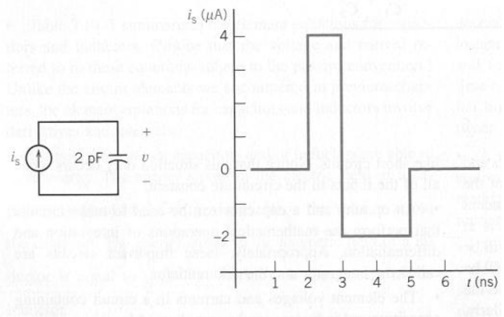
ENGR&204 Spring 2011 Homework #5   Due in one week.

Find v(t) when $v_o(0^-) = -1.0$ mV.



(a)   (b)

for $0 \le t \le 2ns$

note: $V(0^-) = -1mV$
$= V(0^+)$

$$V = \frac{1}{C}\int_{t_i}^{t_f} i_s\,dt + v(t_i)$$

so $$V = \frac{1}{2pF}\int 0\,dt + (-1mV) = -1mV$$

for $2ns \le t \le 3ns$

$$V = \frac{1}{2pF}\int_{2ns}^{t} (4\mu A)dt - 1mV = 2\times10^6\left[t - 2ns\right] - 1mV$$
$$= 2\times10^6 t - 5mV$$

then at $t = 3ns$ $V = 1mV$
this will be the initial voltage for the next time interval.

for $3ns \le t \le 5ns$

$$V = \frac{1}{2pF}\int_{3ns}^{t} (-2\mu A)\,dt + 1mV = -1\times10^6\left[t - 3ns\right] + 1mV$$
$$= -1\times10^6 t + 4mV$$

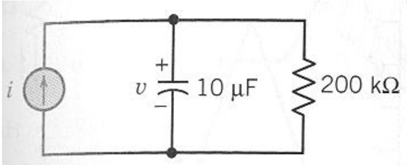
then at $5ns$ $V = -1\,mV$

for $5ns \le t \le \infty$

$$V = \frac{1}{2pF}\int_{5ns}^{\infty} 0\,dt - 1mV = -1mV$$

$$\boxed{V(t) = \begin{cases} -1mV & t \le 0 \\ -1mV & 0 \le t \le 2ns \\ 2\times10^6 t - 5mV & 2ns \le t \le 3ns \\ -1\times10^6 t + 4mV & 3ns \le t \le 5ns \\ -1mV & 5ns \le t \le \infty \end{cases}}$$

Calculate i if $v = 5(1 - 2e^{-2t})$ V



$$i_c = C\frac{dv}{dt}$$

$$= 10\mu F\,\frac{d\left(5(1-2e^{-2t})_V\right)}{dt} = 10\mu F\left((5)4e^{-2t}\,\frac{V}{s}\right)$$

$$= 200e^{-2t}\ \mu A$$

KCL:

$$i = i_C + \frac{V}{200k\Omega}$$

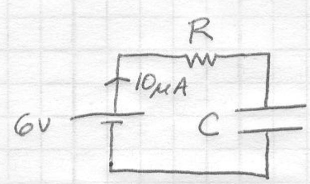
$$= 200e^{-2t}\ \mu A + \frac{5(1-2e^{-2t}\ V)}{200\ k\Omega}$$

$$= 200e^{-2t}\ \mu A + 25\mu A - 50e^{-2t}\ \mu A$$

$$\boxed{= (150e^{-2t} + 25)\mu A}$$

A capacitor is used in a camera electronic flash. A small battery with a constant voltage of 6.0 v is used to charge the capacitor with a constant current of 10 μA. How much time does it take to charge the capacitor when C = 10 μF? What is the stored energy?

The capacitor charging circuit looks like:



at $t=0$ C acts like a short so all 6V is across R. So

$$6V = 10\mu A\,(R)$$

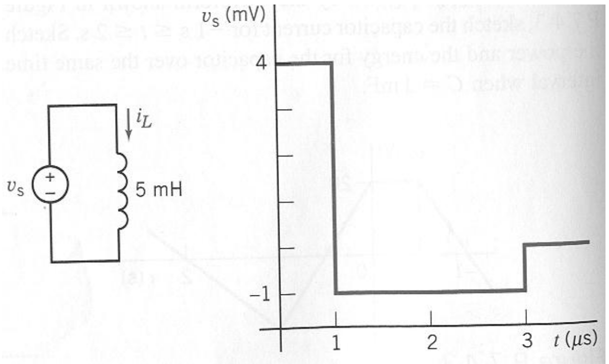
so $R = 600\,k\Omega$

$\tau = RC = (600k\Omega)(10\mu F) = 6s$. It takes about 5τ to get 99.9% charged up. So about $\boxed{30\,s}$ not a very good flash charger!

$W = \frac{1}{2}CV^2 = \frac{1}{2}(10\mu F)(6V)^2 = \boxed{180\mu J}$

C is like an open so no current flows, so $V_c = 6V$.

Find $i_L(t)$ when $t > 0$ when $i_L(0) = -2\mu A$.



$i_L(0^-) = i_L(0^+) = -2\mu A$ use $i = \frac{1}{L}\int_{t_i}^{t_f} v\,dt + i(t_i)$

from $t = 0$ to $1\mu s$

$$i = \frac{1}{5mH}\int_0^t 4mV\,dt - 2\mu A = 800\frac{mA}{s}\left[t - 0\right] - 2\mu A$$
$$= \left(800\frac{mA}{s}\right)t - 2\mu A$$

at $1\mu s$ $i = 800\,nA - 2\mu A$
$= -1.2\,\mu A$

from $1\mu s \le t \le 3\mu s$

$$i = \frac{1}{5mH}\int_{1\mu s}^{t}(-1mV)\,dt - 1.2\mu A = -200\frac{mA}{s}\left[t - 1\mu s\right] - 1.2\mu A$$
$$= -200\frac{mA}{s}\left[t\right] - 1\mu A$$

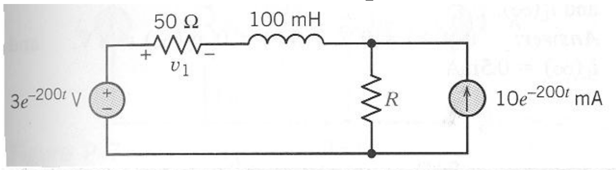
at $3\mu s$ $i = -600\,nA - 1\mu A$
$= -1.6\,\mu A$

from $3ns \le t \le \infty$

$$i = \frac{1}{5mH}\int_{3ns}^{t} 0\,dt - 1.6\mu A = -1.6\mu A$$

$$\boxed{i(t) = \begin{cases} -2\mu A & t \le 0 \\ 800\frac{mA}{s}t - 2\mu A & 0 \le t \le 1\mu s \\ -200t - 1\mu A & 1\mu s \le t \le 3\mu s \\ -1.6\mu A & t \ge 3\mu s \end{cases}}$$

What is R for the circuit if $v_1 = e^{-200t}$ V for $t \geq 0$?



KVL left loop:

$$-3e^{-200t} + e^{-200t} + 100mH \frac{di_L}{dt} + i_R R = 0$$

KCL top node: $i_L + 10e^{-200t}\ mA = i_R$

also $i_L = i_{50\Omega} = \frac{V_1}{50\Omega} = 20mA\, e^{-200t}$

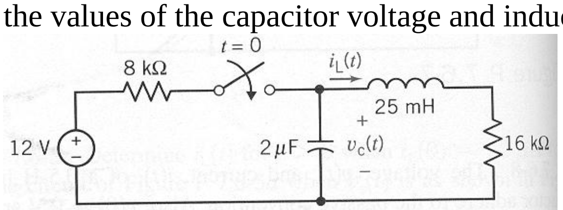
so $\frac{di_L}{dt} = -4e^{-200t}$ so plug all this into KVL eq:

$$-2e^{-200t} + 100mH\left(-4e^{-200t}\right) + \left(20mA\, e^{-200t} + 10mA\, e^{-200t}\right)R = 0$$

$$-2.4e^{-200t} + 30mA\, e^{-200t}(R) = 0$$

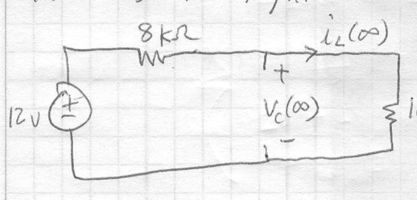
gives $\boxed{R = 80\Omega}$

The switch has been opened for a long time before closing at t = 0 s. What is $v_c(0+)$ and $i_L(0+)$, the values of the capacitor voltage and inductor current immediately after the switch is closed? Find $v_c(\infty)$ and $i_L(\infty)$, the values of the capacitor voltage and inductor current after the switch has been closed for a long while.



Since switch has been open a long time there can't be any current in the inductor and can't be any charge on the capacitor. So $V_c(0^-) = 0 = V_c(0^+)$ and $i_L(0^-) = 0 = i_L(0^+)$.

A long time after the switch is closed the capacitor will be fully charged and will act like an open and the inductor will have lost the fight and will act like a short:



so $i_L(\infty) = \frac{12v}{24k\Omega} = \boxed{500\mu A}$

and $V_c(\infty) = 12v\left(\frac{16k}{16k+8k}\right)$

$= \boxed{8v}$