ENGR& 204 Spring 2010 Homework 5 Key

The voltage at the terminals of the 300 μH inductor is shown. The current i is zero for t ≤ 0. Find an equation for I for t ≥ 0, Sketch i vs t for 0 ≤ t ≤ ∞.

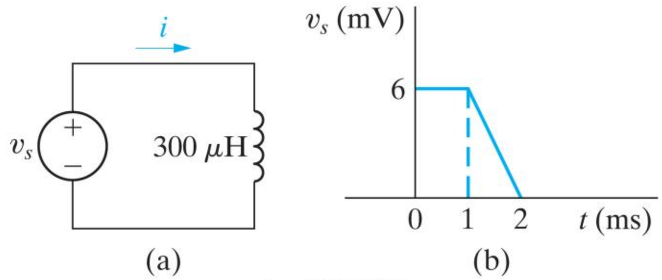


Figure: 06-36-011a,bP6.02

P 6.2 [a] $0 \le t \le 1\,\text{ms}$:

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{300}\int_0^t 6 \times 10^{-3}\,dx + 0$$

$$= 20x \Big|_0^t = 20t\ \text{A}$$

$1\,\text{ms} \le t \le 2\,\text{ms}$:

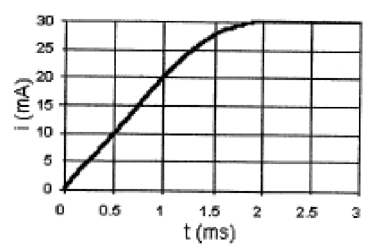
$$i = \frac{10^6}{300}\int_{10^{-3}}^t (12 \times 10^{-3} - 6x)\,dx + 20 \times 10^{-3}$$

$$\therefore \quad i = 40t - 10{,}000t^2 - 10 \times 10^{-3}\ \text{A}$$

$2\,\text{ms} \le t \le \infty$:

$$i = \frac{10^6}{300}\int_{2\times 10^{-3}}^t (0)\,dx + 30 \times 10^{-3} = 30\,\text{mA}$$

[b]



6.8 The current in a 15 mH inductor is 1 A for t ≤ 0 and given by I = A_1e(-2000t) +A_2e(-8000t) A for t ≥ 0. The voltage across the inductor is 60 V at t = 0. Find the voltage across the inductor for t >0, and find the time when the power at the terminals of the inductor is zero.

P 6.8 [a] $i(0) = A_1 + A_2 = 1$

$$\frac{di}{dt} = -2000A_1e^{-2000t} - 8000A_2e^{-8000t}$$

$$v = -30A_1e^{-2000t} - 120A_2e^{-8000t}\text{ V}$$

$$v(0) = -30A_1 - 120A_2 = 60$$

Solving, $A_1 = 2$ and $A_2 = -1$

Thus,

$$i_1 = (2e^{-2000t} - e^{-8000t})\text{ A} \qquad t \geq 0$$

$$v = -60e^{-2000t} + 120e^{-8000t}\text{ V}, \qquad t \geq 0$$

[b] $p = vi = 300e^{-10{,}000t} - 120e^{-4000t} - 120e^{-16{,}000t}$

$p = 0$ when $300e^{6000t} - 120e^{12{,}000t} - 120 = 0$

Let $x = e^{6000t}$; then $300x - 120x^2 - 120 = 0$

Thus $x^2 - 2.5x + 1 = 0$ so $x = 0.5$ and $x = 2$

If $x = e^{6000t} = 0.5$, t will be negative. Hence, the solution for $t > 0$ must be $x = 2$:

$e^{6000t} = 2$ so $6000t = \ln 2$

Thus, $t = \dfrac{\ln 2}{6000} = 115.52\,\mu\text{s}$

6.17 The current shown is applied to a 0.25 μF capacitor with zero initial volts. Find the charge at t = 30 μs, voltage at t = 50 μs, and energy stored in the capacitor by this current.

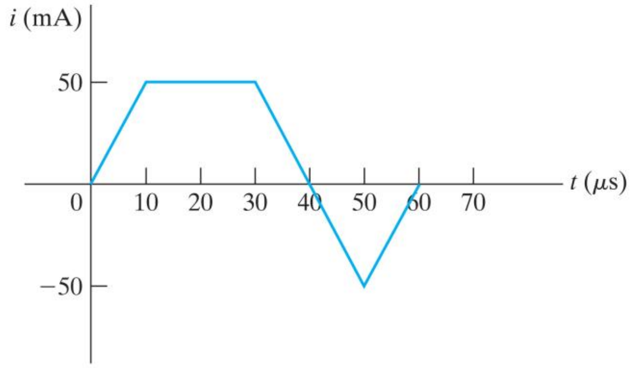


Figure: 06-36-08P6.17

P 6.17 [a] $i = \dfrac{50 \times 10^{-3}}{10 \times 10^{-6}} t = 5 \times 10^3 t \qquad 0 \le t \le 10\,\mu s$

$i = 50 \times 10^{-3} \qquad 10 \le t \le 30\,\mu s$

$$\begin{aligned} q &= \int_0^{10\times10^{-6}} 5 \times 10^3 t\,dt + \int_{10\times10^{-6}}^{30\times10^{-6}} 50 \times 10^{-3}\,dt \\ &= 5 \times 10^3 \frac{t^2}{2}\Big|_0^{10\times10^{-6}} + 50 \times 10^{-3}(20 \times 10^{-6}) \\ &= 5 \times 10^3(\tfrac{1}{2})(100 \times 10^{-12}) + 1000 \times 10^{-3} \times 10^{-6} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

[b] $i = 200 \times 10^{-3} - 5 \times 10^{-3} t \qquad 30\,\mu s \le t \le 50\,\mu s$

$$\begin{aligned} q &= 1.25 \times 10^{-6} + \int_{30\times10^{-6}}^{50\times10^{-6}} [200 \times 10^{-3} - 5 \times 10^3 t]\,dt \\ &= 1.25 \times 10^{-6} + 200 \times 10^{-3}(20 \times 10^{-6}) - 5 \times 10^3 \frac{t^2}{2}\Big|_{30\times10^{-6}}^{50\times10^{-6}} \\ &= 1.25 \times 10^{-6} + 4000 \times 10^{-9} - 5 \times 10^3 \left[\frac{2500 - 900}{2}\right] 10^{-12} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

Since $q = vC, \qquad \therefore \quad v = 1.25/0.25 = 5\,\text{V}.$

[c] $i = -300 \times 10^{-3} + 5 \times 10^{-3} t \qquad 50\,\mu s \le t \le 60\,\mu s$

$$\begin{aligned} q &= 1.25 \times 10^{-6} + \int_{50\times10^{-6}}^{60\times10^{-6}} [-300 \times 10^{-3} + 5 \times 10^3 t]\,dt \\ &= 1.25 \times 10^{-6} - 300 \times 10^{-3}(10 \times 10^{-6}) \\ &\quad + 5 \times 10^3 \left[\frac{3600 - 2500}{2}\right] 10^{-12} \\ &= 1\,\mu\text{C} \end{aligned}$$

$$v = \frac{1 \times 10^{-6}}{0.25 \times 10^{-6}} = 4\,\text{V}$$

$$w = \frac{C}{2} v^2 = \frac{1}{2}(0.25) \times 10^{-6}(16) = 2\,\mu\text{J}$$

6.23 The voltage v_o = 1250e(-25t) V. If $i_1(0)$ = 10 A and $i_2(0)$ = -5 A, find $i_o(0)$, $i_o(t)$ for t≥ 0, $i_1(t)$ for t ≥ 0, $i_2(t)$ for t ≥ 0, the initial energy stored in the three inductors, the total energy delivered to the black box, and the energy trapped in the ideal inductors.

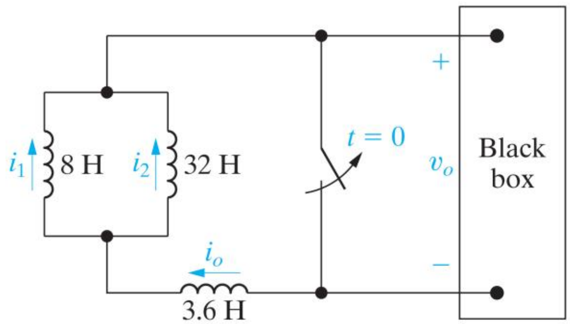
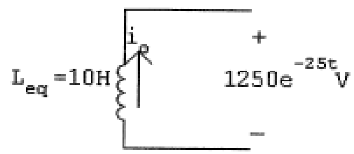


Figure: 06-36-14P6.23

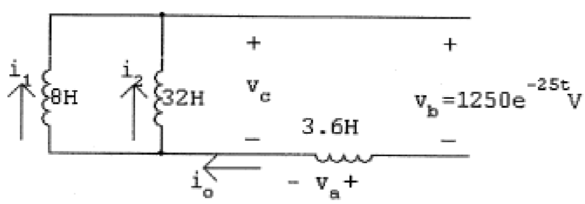
P 6.23 [a] $i_o(0) = i_1(0) + i_2(0) = 5\,\text{A}$

[b]



$$i_o = -\frac{1}{10}\int_0^t 1250e^{-25x}\,dx + 5 = -125\left[\frac{e^{-25x}}{-25}\Big|_0^t\right] + 5$$
$$= 5(e^{-25t} - 1) + 5 = 5e^{-25t}\,\text{A}, \qquad t \ge 0$$

[c]



$$v_a = 3.6\frac{d}{dt}(5e^{-25t}) = -450e^{-25t}\,\text{V}$$

$$v_c = v_a + v_b = -450e^{-25t} + 1250e^{-25t}$$
$$= 800e^{-25t}\,\text{V}$$
$$i_1 = -\frac{1}{8}\int_0^t 800e^{-25x}\,dx + 10$$
$$= 4e^{-25t} - 4 + 10$$
$$i_1 = 4e^{-25t} + 6\,\text{A} \qquad t \ge 0$$

[d] $$i_2 = -\frac{1}{32}\int_0^t 800e^{-25x}\,dx - 5$$
$$= e^{-25t} - 1 - 5$$
$$i_2 = e^{-25t} - 6\,\text{A}, \qquad t \ge 0$$

[e] $w(0) = \frac{1}{2}(8)(100) + \frac{1}{2}(32)(25) + \frac{1}{2}(3.6)(25) = 845\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(10)(25) = 125\,\text{J}$

[g] $w_{\text{trapped}} = 845 - 125 = 720\,\text{J}$

7.8 The switch has been closed a long time. At t = 0 it is opened. Find $v_o(t)$ for $t \ge 0$.

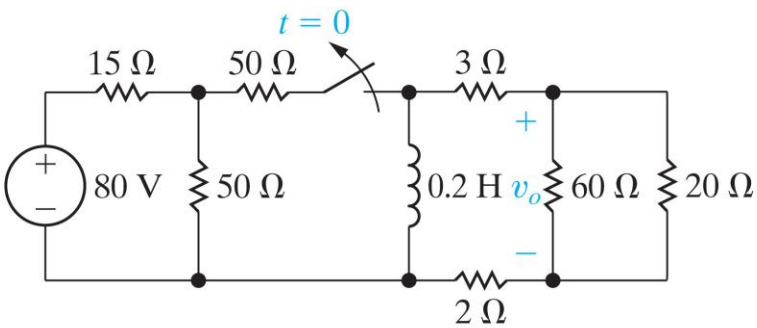
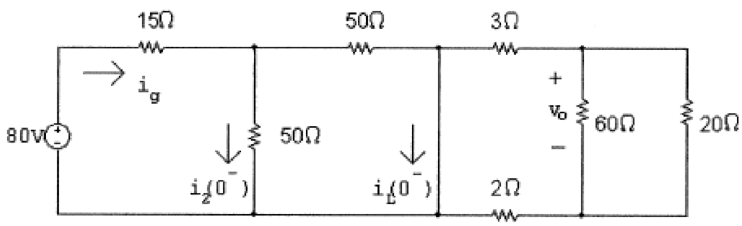


Figure: 07-48-07P7.08

Note: use the cookbook!

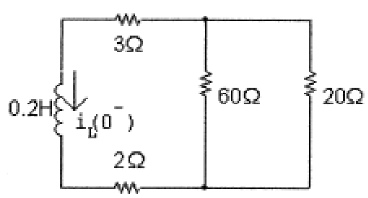
P 7.8 [a] For $t < 0$



$$i_g = \frac{80}{40} = 2\,\mathrm{A}$$

$$i_L(0^-) = \frac{2(50)}{(100)} = 1\,\mathrm{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\mathrm{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.20}{5+15} = \frac{1}{100} = 0.01\,\mathrm{s}$$

$$i_L(0^+) = 1\,\mathrm{A}$$

$$i_L(t) = e^{-100t}\,\mathrm{A}, \qquad t \geq 0$$

$$v_o(t) = -15 i_L(t)$$

$$v_o(t) = -15e^{-100t}\,\mathrm{V}, \qquad t \geq 0^+$$

7.14 The switch has been closed a long time and opens at t =0. Find $v_o(t)$ for $t \geq 0^+$

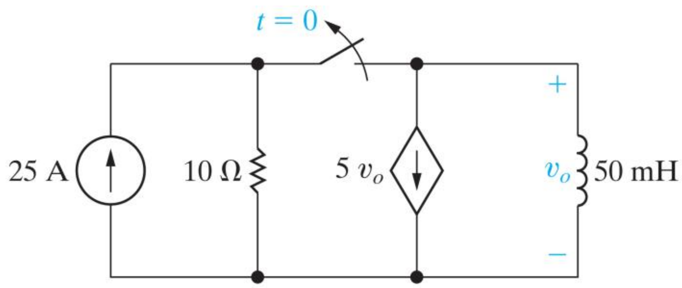
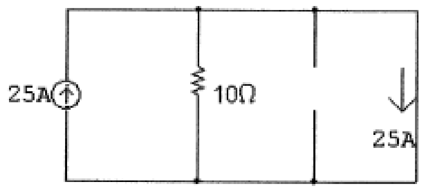


Figure: 07-48-11P7.14

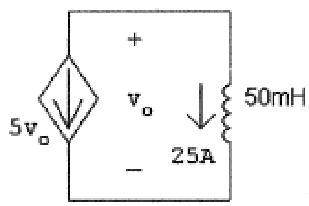
Note: use the cookbook!

P 7.14 $t < 0$

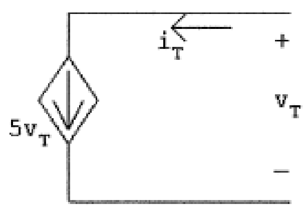


$$i_L(0^-) = i_L(0^+) = 25\,\text{A}$$

$t > 0$

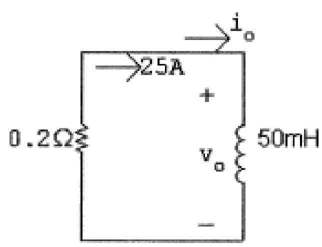


Find Thévenin resistance seen by inductor



$$i_T = 5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{5} = 0.2\,\Omega$$

$$\tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{0.2} = 250\,\text{ms}; \qquad 1/\tau = 4$$



$$i_o = 25e^{-4t}\,\text{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o}{dt} = (50 \times 10^{-3})(-100e^{-4t}) = -5e^{-4t}\,\text{V}, \quad t \geq 0^+$$