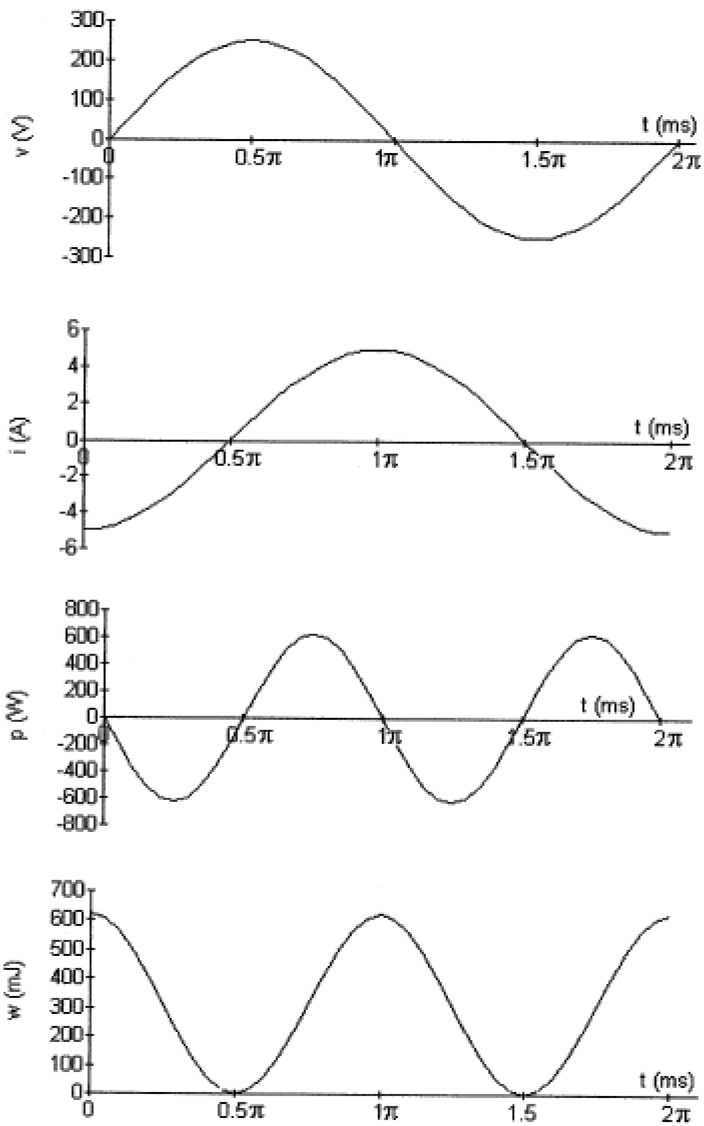
ENGR& 204 Homework 5 key Spring

P 6.7 [a]

$$\begin{aligned} i &= \frac{1000}{50}\int_0^t 250\sin 1000x\,dx - 5 \\ &= 5000\int_0^t \sin 1000x\,dx - 5 \\ &= 5000\left[\frac{-\cos 1000x}{1000}\right|_0^t - 5 \\ &= 5(1-\cos 1000t) - 5 \\ i &= -5\cos 1000t\ \text{A} \end{aligned}$$

2009

$$
\begin{aligned}
\textbf{[b]} \quad p &= vi = (250\sin 1000t)(-5\cos 1000t) \\
&= -1250\sin 1000t\cos 1000t \\
p &= -625\sin 2000t \text{ W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(50\times 10^{-3})25\cos^2 1000t \\
&= 625\cos^2 1000t \text{ mJ} \\
w &= [312.5 + 312.5\cos 2000t] \text{ mJ.}
\end{aligned}
$$



[c] Absorbing power: Delivering power:

Absorbing power:	Delivering power:
$0.5\pi \leq t \leq \pi$ ms	$0 \leq t \leq 0.5\pi$ ms
$1.5\pi \leq t \leq 2\pi$ ms	$\pi \leq t \leq 1.5\pi$ ms

P 6.10 $i = (B_1 \cos 5t + B_2 \sin 5t)e^{-t}$

$$i(0) = B_1 = 25\,\text{A}$$

$$\frac{di}{dt} = (B_1 \cos 5t + B_2 \sin 5t)(-e^{-t}) + e^{-t}(-5B_1 \sin 5t + 5B_2 \cos 5t)$$

$$= [(5B_2 - B_1)\cos 5t - (5B_1 + B_2)\sin 5t]e^{-t}$$

$$v = 2\frac{di}{dt} = [(10B_2 - 2B_1)\cos 5t - (10B_1 + 2B_2)\sin 5t]e^{-t}$$

$$v(0) = 100 = 10B_2 - 2B_1 = 10B_2 - 50 \quad \therefore \quad B_2 = 150/10 = 15\,\text{A}$$

Thus,

$$i = (25\cos 5t + 15\sin 5t)e^{-t}\,\text{A}, \qquad t \geq 0$$

$$v = (100\cos 5t - 280\sin 5t)e^{-t}\,\text{V}, \qquad t \geq 0$$

$$i(0.5) = -6.70\,\text{A}; \qquad v(0.5) = -150.23\,\text{V}$$

$$p(0.5) = (-6.70)(-150.23) = 1007.00\,\text{W absorbing}$$

P 6.16 [a] $i = C\frac{dv}{dt} = 0, \quad t < 0$

[b] $i = C\frac{dv}{dt} = 5e^{-1000t}[\cos 3000t + 13\sin 3000t]\,\text{mA}, \quad t \geq 0$

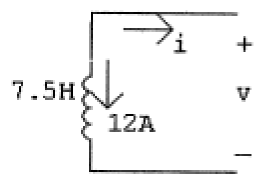
[c] no, $v(0^-) = -30$ V
$v(0^+) = 10 - 40 = -30$ V

[d] yes, $i(0^-) = 0$ A
$i(0^+) = 5$ mA

[e] $v(\infty) = 10\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.5 \times 10^{-6})(10)^2 = 25\,\mu\text{J}$$

P 6.22 [a]



$$\begin{aligned} i(t) &= -\frac{1}{7.5}\int_0^t -1800e^{-20x}\,dx - 12 \\ &= 240\frac{e^{-20x}}{-20}\Big|_0^t -12 \\ &= -12(e^{-20t} - 1) - 12 \\ i(t) &= -12e^{-20t}\,\text{A} \end{aligned}$$

[b] $$\begin{aligned} i_1(t) &= -\frac{1}{10}\int_0^t -1800e^{-20x}\,dx + 4 \\ &= 180\frac{e^{-20x}}{-20}\Big|_0^t + 4 \\ &= -9(e^{-20t} - 1) + 4 \\ i_1(t) &= -9e^{-20t} + 13\,\text{A} \end{aligned}$$

[c] $$\begin{aligned} i_2(t) &= -\frac{1}{30}\int_0^t -1800e^{-20x}\,dx - 16 \\ &= 60\frac{e^{-20x}}{-20}\Big|_0^t - 16 \\ &= -3(e^{-20t} - 1) - 16 \\ i_2(t) &= -3e^{-20t} - 13\,\text{A} \end{aligned}$$

[d] $p = vi = (-1800e^{-20t})(-12e^{-20t}) = 21{,}600e^{-40t}\,\text{W}$

$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 21{,}600e^{-40t}\,dt \\ &= 21{,}600\frac{e^{-40t}}{-40}\Big|_0^\infty \\ &= 540\,\text{J} \end{aligned}$$

[e] $w = \frac{1}{2}(10)(16) + \frac{1}{2}(30)(256) = 3920\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 3920 - 540 = 3380\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(10)(13)^2 + \frac{1}{2}(30)(13)^2 = 3380\,\text{J}$ checks

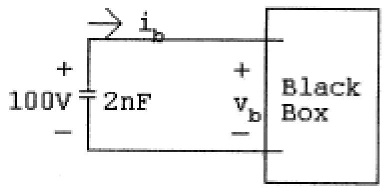
P 6.28 $C_1 = 1 + 1.5 = 2.5\,\text{nF}$

$$\frac{1}{C_2} = \frac{1}{2.5} + \frac{1}{12.5} + \frac{1}{50} = \frac{1}{2}$$

$\therefore \quad C_2 = 2\,\text{nF}$

$v_\text{d}(0) + v_\text{a}(0) - v_\text{c}(0) = 40 + 15 + 45 = 100\,\text{V}$

[a]



$$\begin{aligned} v_\text{b} &= -\frac{10^9}{2}\int_0^t 50 \times 10^{-6} e^{-250x}\,dx + 100 \\ &= -25{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 100 \\ &= 100(e^{-250t} - 1) + 100 \\ &= 100e^{-250t}\,\text{V} \end{aligned}$$

[b] $$v_a = -\frac{10^9}{12.5}\int_0^t 50\times10^{-6}e^{-250x}\,dx + 15$$

$$= -4000\frac{e^{-250x}}{-250}\Big|_0^t + 15$$

$$= 16(e^{-250t}-1)+15$$

$$= 16e^{-250t} - 1\,\text{V}$$

[c] $$v_c = \frac{10^9}{50}\int_0^t 50\times10^{-6}e^{-250x}\,dx - 45$$

$$= 1000\frac{e^{-250x}}{-250}\Big|_0^t - 45$$

$$= -4(e^{-250t}-1)-45$$

$$= -4e^{-250t} - 41\,\text{V}$$

[d] $$v_d = -\frac{10^9}{2.5}\int_0^t 50\times10^{-6}e^{-250x}\,dx + 40$$

$$= -20{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 40$$

$$= 80(e^{-250t}-1)+40$$

$$= 80e^{-250t} - 40\,\text{V}$$

CHECK: $$v_b = v_d + v_a - v_c$$

$$= 80e^{-250t} - 40 + 16e^{-250t} - 1 + 4e^{-250t} + 41$$

$$= 100e^{-250t}\,\text{V} \quad \text{(checks)}$$

[e] $$i_1 = -10^{-9}\frac{d}{dt}[80e^{-250t}-40]$$

$$= -10^{-9}(-20{,}000e^{-250t})$$

$$= 20e^{-250t}\,\mu\text{A}$$

[f] $$i_2 = -1.5\times10^{-9}\frac{d}{dt}[80e^{-250t}-40]$$

$$= -1.5\times10^{-9}(-20{,}000e^{-250t})$$

$$= 30e^{-250t}\,\mu\text{A}$$

CHECK: $i_1 + i_2 = 50e^{-250t}\,\mu\text{A} = i_b$

P 6.33
$$\begin{aligned} v_c &= \frac{-10^6}{20}\int_0^t e^{-80x}\sin 60x\,dx - 300 \\ &= 5e^{-80t}[80\sin 60t + 60\cos 60t] + 300 - 300 \\ &= 400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t \text{ V} \\ v_L &= 5\frac{di_o}{dt} \\ &= 5[-80e^{-80t}\sin 60t + 60e^{-80t}\cos 60t] \\ &= -400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t \text{ V} \\ v_o &= v_c - v_L \\ &= (300e^{-80t}\cos 60t - 300e^{-80t}\cos 60t + 400e^{-80t}\sin 60t + \\ &\quad 400e^{-80t}\sin 60t) \\ &= 800e^{-80t}\sin 60t \text{ V} \end{aligned}$$

P 7.5
$$w(0) = \frac{1}{2}(20\times 10^{-3})(10^2) = 1\text{ J}$$

$$0.5w(0) = 0.5\text{ J}$$

$$i_R = 10e^{-t/\tau}$$

$$p_{\text{diss}} = i_R^2 R = 100Re^{-2t/\tau}$$

$$w_{\text{diss}} = \int_0^t R(100)e^{-2x/\tau}\,dx$$

$$w_{\text{diss}} = 100R\frac{e^{-2x/\tau}}{-2/\tau}\Big|_0^{t_o} = -50\tau R(e^{-2t_o/\tau} - 1) = 50L(1 - e^{-2t_o/\tau})$$

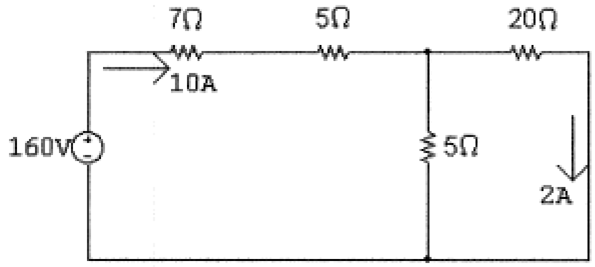
$$50L = (50)(20)\times 10^{-3} = 1; \qquad t_o = 10\,\mu\text{s}$$

$$1 - e^{-2t_o/\tau} = 0.5$$

$$e^{2t_o/\tau} = 2; \qquad \frac{2t_o}{\tau} = \frac{2t_oR}{L} = \ln 2$$

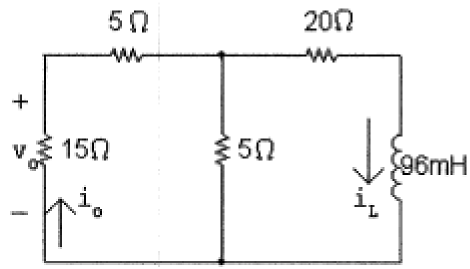
$$R = \frac{L\ln 2}{2t_o} = \frac{20\times 10^{-3}\ln 2}{20\times 10^{-6}} = 693.15\,\Omega$$

P 7.11 $t < 0$:



$i_L(0^+) = 2\,\text{A}$

$t > 0$:



$$R_e = \frac{(20)(5)}{25} + 20 = 24\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{96}{24} \times 10^{-3} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

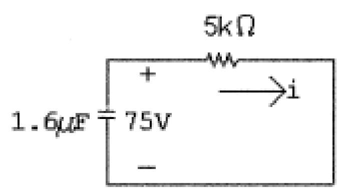
$$\therefore \quad i_L = 2e^{-250t}\,\text{A}$$

$$\therefore \quad i_o = \frac{5}{25} i_L = 0.4e^{-250t}\,\text{A}$$

$$v_o = -15i_o = -6e^{-250t}\,\text{V}, \quad t \geq 0^+$$

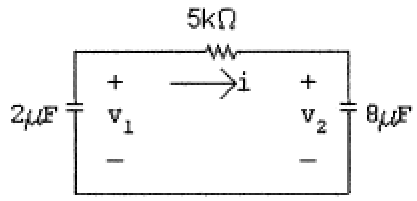
P 7.21 **[a]** $v_1(0^-) = v_1(0^+) = 75\,\text{V}$ $\qquad v_2(0^+) = 0$

$C_{\text{eq}} = 2 \times 8/10 = 1.6\,\mu\text{F}$



$$\tau = (5)(1.6) \times 10^{-3} = 8\text{ms}; \qquad \frac{1}{\tau} = 125$$

$$i = \frac{75}{5} \times 10^{-3} e^{-125t} = 15e^{-125t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-10^6}{2} \int_0^t 15 \times 10^{-3} e^{-125x}\,dx + 75 = 60e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{10^6}{8} \int_0^t 15 \times 10^{-3} e^{-125x}\,dx + 0 = -15e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

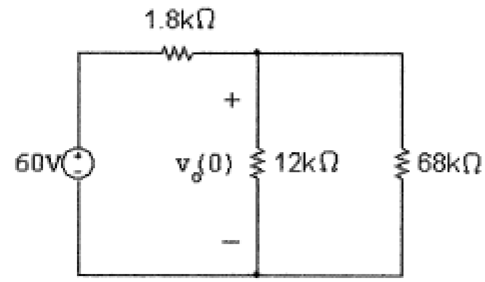
[b] $w(0) = \frac{1}{2}(2 \times 10^{-6})(5625) = 5625\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(225) + \frac{1}{2}(8 \times 10^{-6})225 = 1125\,\mu\text{J}.$

$$w_{\text{diss}} = \frac{1}{2}(1.6 \times 10^{-6})(5625) = 4500\,\mu\text{J}.$$

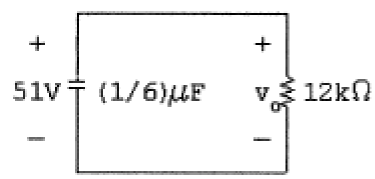
Check: $w_{\text{trapped}} + w_{\text{diss}} = 1125 + 4500 = 5625\,\mu\text{J}; \qquad w(0) = 5625\,\mu\text{J}.$

P 7.26 [a] $t < 0$:



$$v_o(0) = \frac{(60)(10.2)}{12} = 51\,\text{V}$$

$t > 0$:



$$\tau = \frac{1}{6}(12) \times 10^{-3} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 51e^{-500t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12} \times 10^{-3} = 216.75 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{2\times 10^{-3}} 216.75 \times 10^{-3}e^{-1000t}\,dt$$
$$= 216.75 \times 10^{-6}(1 - e^{-2}) = 187.42\,\mu\text{J}$$

[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)(51)^2 \times 10^{-6} = 216.75\,\mu\text{J}$

$$0.95w(0) = 205.9125\,\mu\text{J}$$

$$\int_0^{t_o} 216.75 \times 10^{-3}e^{-1000x}\,dx = 205.9125 \times 10^{-6}$$

$$\int_0^{t_o} e^{-1000x}\,dx = 0.95 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-1000t_o} = 0.95; \qquad e^{1000t_o} = 20; \quad \text{so} \quad t_o = 3\,ms$$